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SPRING DRIVEN PRIMER STRIKER STUDY

Gerard G. Lowen

City College Research Foundation

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FINAL REPORT

By

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This research was conducted under contract No. DAAG-39 -68-C-0026 with the Harry Diamond Laboratories, Branch 420. Washington, D. C. 20438

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List of Symbols

A	=	cross-sectional area of bar or spring wire (in^2)
a	=	velocity of wave propagation or surge velocity (in/sec)
α	=	m_s/M, the ratio of mass of spring to the driven mass
c	=	D/d, the spring index, ratio of coil to wire diameter
c'	=	Spring constant per coil of spring
C_f	=	coefficient of expansion
C_D	=	design correction factor
d	=	wire diameter of spring (in)
D	=	mean coil diameter of spring (in)
D_o	=	outside diameter of firing pin and spring
E	=	modulus of elasticity (lbs/in^2)
E_K	=	kinetic energy (in-lbs)
E_s	=	strain energy (in-lbs)
E_{su}	=	strain energy per unit volume ($in\text{-}lbs/in^3$)
ε	=	strain associated with stress σ (in/in)
f	=	total spring deflection (in)
f'	=	deflection of spring per active coil (in)
F	=	contact force between mass and spring (lbs)
F_t	=	total distance travelled by mass M between $t = 0$ and $t = T_A$ (in)
γ	=	ρg, density (lbs/in^3)
g	=	gravitational constant ($386.05\ in/sec^2$)
G	=	modulus of rigidity (lbs/in^2)
J_o	=	area polar moment of inertia of spring wire (in^4)
k	=	spring constant per unit length of bar or spring wire (lbs /in/in)

List of Symbols (cont.)

K = curvature correction factor for helical springs

L = length of bar or spring wire (in)

L_{ex} = Length of spring when firing pin attains max. velocity

L_o = free length of spring (in)

L_p = length of firing pin (in)

L_t = overall required system length to attain design velocity (in)

λ = spring constant of helical spring (lbs/in)

m = mass per unit length of bar or spring wire (lb-sec^2/in^4)

m_s = mass of active coils of spring (lb-sec^2/in)

M = mass of driven mass (firing pin) (lb-sec^2/in)

M_D = d'Alembert moment

N = number of active coils of helical spring

N_t = total number of coils of helical spring

P = applied force (lbs)

P_f = force due to deflection f (lbs)

ρ = mass density (lb-sec^2/in^4)

σ = compressive or tensile stress (lbs/in^2)

T_D = D'Alembert force

t = time (sec)

T_s = surge time

T = $2\ T_s$

T_z = time at which spring flies off support, counted from spring release (sec)

T_A = time of separation of mass from spring, counted from spring release (sec)

τ_f = uncorrected shear stress corresponding to deflection f, (lbs/in^2)

τ_c = $K\tau_f$ corrected shear stress corresponding to deflection f, (lbs/in^2)

v = particle velocity (in/sec)

v_H = maximum velocity attainable by freely expanding spring without end load as it flies off support (in/sec)

v_T = v_{max}, the maximum theoretically attainable velocity of the driven mass (firing pin) for a given system and spring compression (in/sec)

v_N = nominal firing pin velocity (in/sec)

W = work (in-lbs)

W_P = work due to force P (in-lbs)

Abstract

A space optimization procedure is developed which allows the design of 100 percent reliable spring-firing pin combinations for the M42G percussion primer. Optimum design tables list complete data for approximately two hundred systems ranging in overall length from 0.500 to 2.000 inches and in diameter from 0.093 to 0.375 inches.

The optimization is based on a combination of results of the theoretical and experimental phases of the investigation. The theoretical work employs wave propagation theory for the determination of the maximum attainable velocities of spring driven masses. The experimental work confirmed existing 100 percent firing data (Erwood curves) for the M42G primer. In addition, new 100 percent firing data were obtained, at various energy levels, for firing pin velocities up to 1200 inches per second. All firing tests were conducted at ambient temperature and at -40 degrees Fahrenheit.

I. Introduction

1. Purpose of Investigation

The present investigation had the following two aims:

a. It was desired to confirm and to extend to higher firing pin velocities the existing firing curves (Erwood Curves) for the M42G percussion primer. This experimental investigation was to be conducted both at ambient temperature as well as at -40 degrees F.

b. It was further desired to devise a design method which furnishes 100 percent reliable spring driven primer striker systems with minimum overall space requirements for the M42G primer.

2. Outline and Results of Investigation

a. M42G percussion primers were test-fired with firing pin velocities ranging from 50 to 1200 inches per second at energy levels of 16,14,12, 10, 8, and 6 inch-ounces. These firings took place both at ambient temperature and at -40 degrees F.

The Erwood Curves were partially confirmed. That is, one hundred percent firing of the primers at ambient temperature started at the same minimum firing pin velocities for energy levels of 16 and 14 inch-ounces. For 12, 10, 8, and 6 inch-ounce levels the minimum firing pin velocities for 100 percent firing were somewhat higher than those of the corresponding Erwood data. Increases of the velocities beyond the above minimum values and up to 1200 inches per second caused consistent 100% firings at all energy levels. While in most cases a slightly higher minimum velocity was required to fire the primers at -40 degrees F, one may generally state that there is no significant difference between firings at ambient temperature and those at -40 degrees F. The attempt to conduct firing tests at an energy level of 5 inch-ounces was not successful.

b. The theoretical phase of the investigation concerned itself with the determination of an expression for the maximum attainable velocity of a spring driven mass in the absence of friction. Existing earlier work was confirmed to this end. In addition, an expression for the axial space need of a helical spring which accelerates a mass to its maximum attainable velocity was derived.

c. An optimization procedure was devised which combines experimental and theoretical results. It allows the design of spring striker combinations of minimum overall lengths and practical diameters which employ identical firing pin masses and produce identical firing pin velocities as systems which have been found one hundred percent reliable during the test phase of the investigation.

The overall length of the spring striker system, i.e. the combined lengths of the firing pin and the length of the helical spring when imparting the prescribed velocity to the firing pin, has been found to be a function of the spring index (the ratio of the coil diameter to the wire diameter).

Computer searches for the optimum spring index for various combinations have resulted in optimum design tables. These tables contain all necessary physical data for springs and firing pins and overall dimensions for more

than two hundred systems with overall lengths of less than 2.000 inches and diameters of less than 0.375 inches. (One of the smallest designs requires for a diameter of 0.125 inches a total nominal length of 0.763 inches.)

Sections II to IV give the highlights of all work which is described in detail in Appendices A to E. The use of the optimum design tables of Appendix F is also illustrated in section IV.

During the initial phase of the investigation the literature dealing with percussion primers and explosive initiation [1 - 15]*1, as well as that dealing with penetration of plates by impacting bodies [16-24], was examined with the intent of establishing some theoretical criteria concerning firing pin masses and velocities required to effect primer firing. This effort proved fruitless and had to be abandoned because of the scarcity of information on percussion primer initiation.

II. Theoretical Investigation

The following outlines the results of the theoretical investigation which are described in detail in Appendices A, B, and C. Figure 1 shows a helical spring of coil diameter D and circular wire diameter d which is fixed on its left end Z to the support S, while its right end A is deflected through the distance f by

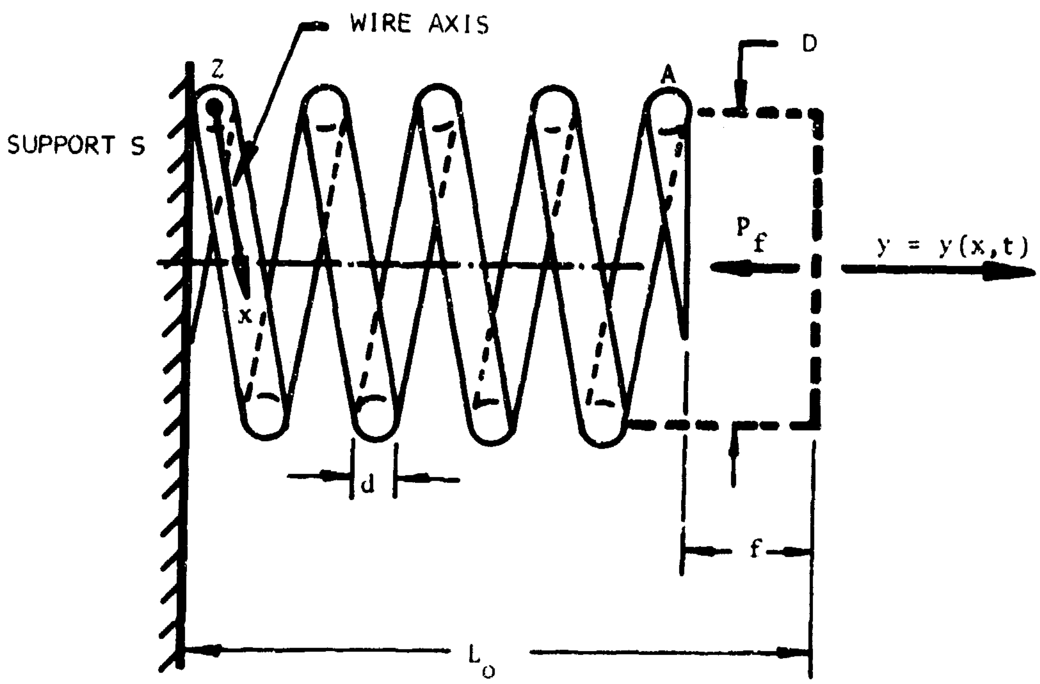


Figure 1

Helical Spring Without Driven Mass
(Deflected through distance f by force P_f)

force P_f. The free length of the spring is L_o. Appendix A gives the derivation of the partial differential equation for the axial displacement y as a function of the position x along the wire axis and of time.

*1 Numbers in brackets designate references in Section V.

(See also [32,33,40, 42]). The constant coefficient of the differential equation A-16 contains an expression for the velocity of wave propagation, or surge velocity a, with which a relieving tension wave moves from end A of the spring wire axis to end Z when the force P_f is suddenly removed. The discussion preceding Eq. (A-19) shows that whenever the spring index D/d>5, the influence of the rotational inertia of the spring wire will sufficiently accounted for in the following expression for the surge velocity:

$$a = 0.99 \sqrt{\frac{\lambda\ L\ g}{\gamma\ (\frac{\pi\ d^2}{4})}} \tag{1}$$

where

λ = spring constant [see Eq. (A-1a)]

L = length of spring wire along x-axis, and embracing all active turns

γ = density of spring material

g = acceleration of gravity

[See also List of Symbols]

The solution of the partial differential equation indicates that the maximum velocity of the free end of the spring, (point A), is attained according to Eq. (A-41) at the time

$$t = T_s = \frac{L}{a} \tag{2}$$

after the force P_f has been removed. T_s is defined as the surge time. This maximum velocity v_H of the expanding helical spring is given by Eq. (A-48):

$$v_H = 0.99\ \tau_f \sqrt{\frac{g}{2\ \gamma\ G}} \tag{3}$$

where

τ_f = uncorrected shear stress associated with deflection f [See Eq. (A-45)]

G = modulus of rigidity

Equation (3) points up the fact that the maximum velocity attainable by the free end of a given spring is only a function of the shear stress due to its deflection. This velocity is thus clearly limited by the maximum permissible uncorrected shear stress.[2]

---

[2]The corrected shear stress $\tau_c = K\ \tau_f$ includes the effect of the curvature correction factor K, (see [42] and Eq. (D-8) in Appendix D). It must be considered to avoid spring set. Therefore τ_f must be low enough so that τ_c does not predict set.

[See also Eq. (A-49) and subsequent discussion on theoretical velocity limit for steel helical springs.]

When the spring drives a mass M, as shown in Figure 2, the maximum velocity of point A, and with that of the mass M, can also be obtained by way of the solution of an appropriate partial differential equation. Since a separate numerical determination of the eigen-values must be made for each occurring

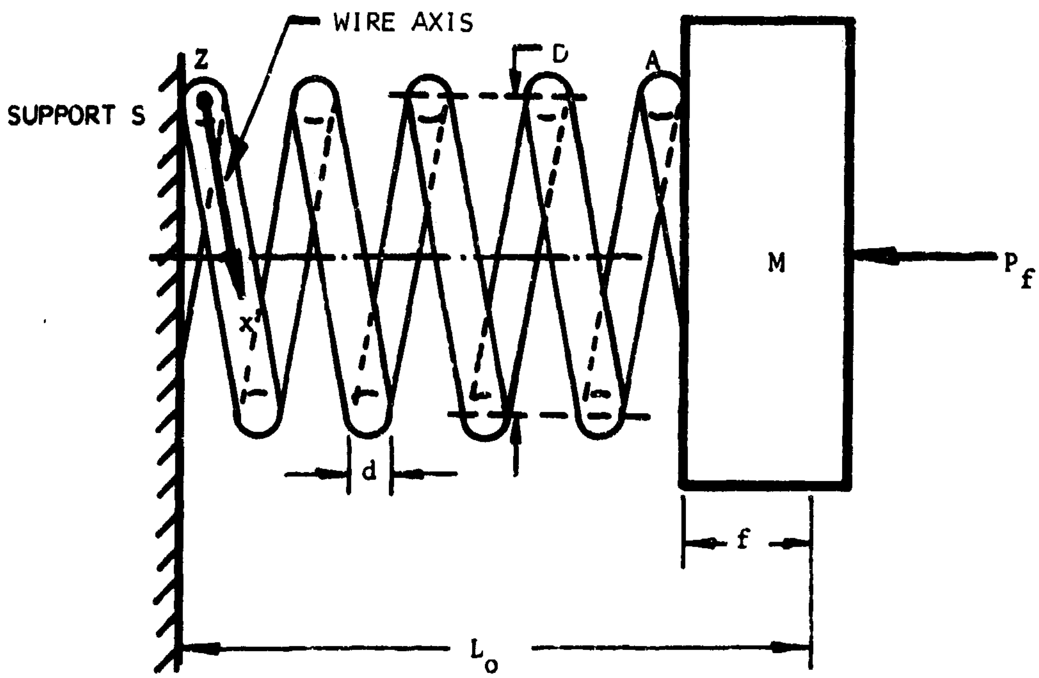


Figure 2

Helical Spring With Driven Mass
(Deflected through distance f by force P_f)

ratio of driven mass M to spring mass m_s, this approach is not very practical and was not further pursued. In addition, it is limited to cases where the end Z of the spring remains fixed to the support S at all times, while mass M is firmly attached to the spring. In fuze applications it is not customary to attach the spring to its support and the mass to the spring. Thus if enough room is given the spring may fly off of its support and the mass may separate from the spring.

K. Maier [28,35,36,37] has shown without proof that wave propagation theory furnishes relatively simple analytical expressions for the behavior of spring driven masses. Beyond this it allows for the possible separation of the spring from its support as well as the separation of the mass from the spring.

Appendix B gives derivations of appropriate expressions along these lines. Starting from the basic concepts of wave propagation in thin prismatical bar [26,31,39,41] the response of springs without driven end masses is considered first. (See also [25,27,29,30,34,38,43]). Equations (B-25) and (B-26) indicate that as the relieving tensile wave, which is initiated by the removal of the compressive force P_f, travels from point A to point Z (see Figure 1), the particles along the wire axis acquire the velocity v_H as given by Eq. (3). Once point Z near the support has attained the above velocity the spring will fly off from the support. This will occur at the time T_s after the force is removed. [T_s is also given by Eq. (2) above.]

For the determination of the velocity of the spring driven mass of Figure 2 it is necessary to distinguish between two time intervals of duration $2T_s$, since the wave behavior in each of them differs from the other. Appendix B gives detailed discussions of these phenomena.

When the compressive force P_f is suddenly removed at t = 0, the velocity of mass M, for the interval $0 \leq t < 2T_s$, is given according to Eq. (B-50b) by:

$$v_1 = v_H \left(1 - e^{-\frac{2\alpha}{T} t} \right) \tag{4}$$

where

$T = 2\,T_s$

$\alpha = m_s/M$

m_s = mass of the active coils of the helical spring

Eq. (4) is valid for all values of the mass ratio α.

The maximum velocity of mass M is attained during the interval $2T_s \leq t < 4T_s$ for values of $1/\alpha \leq 5.69$. Since this range of mass ratios covers all possible applications to primer systems, no other intervals were considered.

The maximum velocity occurs when the contact force between mass M and the spring vanishes, and the mass separates from the spring. Equation (B-82) gives the time of this event for a given system as

$$T_A = T_s \left(2 + \frac{1}{2\alpha} e^{-2\alpha} \right) \tag{5}$$

after the compressive force is suddenly removed. The mass ratio restriction mentioned above is shown by Eq. (B-86) to be based on the condition that $T_A < 4T_s$.

Equation (B-84) furnishes the expression for the velocity of the mass in the time interval $2T_s \leq t < T_A$:

$$v_2 = v_H[e^{-2\alpha(\frac{t}{T} - 1)} (\frac{4\alpha}{T} + 2 - 4\alpha - 2e^{-2\alpha}) - 1] \tag{6}$$

The maximum attainable velocity of the mass M is given by the evaluation of Eq. (6) at $t = T_A$, as given by Eq. (5). According to Eq. (B-85):

$$v_{max} = v_H (- 1 + 2e^{-\frac{e^{-2\alpha}}{2}}) \tag{7}$$

Figure B-7 on page B-23 of Appendix B lists values of the term in parenthesis of Eq. (7) for mass ratios M/m_s from 0 to 5.4.

Appendix C shows how Eq's. (4) and (6) may be integrated to obtain the total distance F_t travelled by mass M from the time the restraining force P_f is suddenly removed at $t = 0$ until the maximum attainable velocity is reached at $t = T_A$. Equation (C-13) gives this distance in the following form:

$$F_t = f\, C_f \tag{8}$$

where

f = deflection due to the force P_f, and

$$C_f = 2 + \frac{1}{\alpha} (3 - \frac{e^{-2\alpha}}{2} - 4\, e^{-\frac{e^{-2\alpha}}{2}}) \tag{9}$$

The mass ratio restriction $M/m_s \leq 5.69$ also applies to the above, since Eq. (6) is involved in its derivation.

Equation (7) which allows the determination of the maximum velocity of mass M for a given system, and Eq. (8) which indicates the minimum space needs to attain this maximum velocity play a major part in the optimization procedure shown in section IV.

Appendix B also indicates, (see pp. B-15 and B-16), how wave propagation theory may be used for the determination of the time T_Z at which a spring which drives a mass will separate from its support at point Z.

III Experimental Investigation

Appendix D reports in detail on all phases of the test program, test setup, and test results.

Figure D-2 shows the original test program, giving all planned test points at 16,14,12,10,8,6, and 5 inch-ounces of firing pin kinetic energy. The associated firing pin masses and velocities, from 50 inches per second to 1200 in/sec, are indicated together with the respective Identification Numbers. Figure D-3 depicts the test setup. The test firing pins are described in Figures D-4 to D-8c, and the final design of the test springs is shown to be based on the results of the theoretical investigation. The physical data of these springs are listed in Figure D-9. In addition, the two methods of measuring firing pin velocity are discussed together with all test procedures. Figures D-12a and b report on the number of firings at the various test points, both at ambient temperature and at -40 degrees F, while Figure D-13 represents a typical data sheet.

1. Results of Firing Tests

Figure 3 gives the results of all firing tests, which were completed successfully, by indicating the percentage of primers which fired at each energy level, firing pin velocity, and temperature (The test results within the heavy outlines serve as the basis of the Optimum Design Tables of Appendix F.) The 5 inch-ounce tests proved to be too inconsistent, and were not completed.

The Erwood data were only partially confirmed. A comparison of Figure D-1, the original Erwood Curves (which refer to ambient temperature), with the comparable data of Figure 3 indicates that for energy levels of 16 and 14 inch-ounces both tests showed that 100 percent firing of the primers starts at a minimum firing pin velocity of 100 in/sec.

For energy levels of 12 inch-ounces and less, the present tests show somewhat higher minimum firing pin velocities for 100 percent firing at ambient temperature than the Erwood data. For 12 inch-ounces the Erwood data give a minimum velocity of approximately 100 in/sec, while the present test required 200 in/sec. At 10 inch-ounces the comparable velocities are approximately 140 and 200 in/sec. The Erwood data show 260 in/sec for 9 in-oz, while present tests indicate for 8 in-oz a reliable value at 500 in/sec. Finally, at 6 in-oz the comparable numbers are approximately 340 in/sec for the Erwood data and 600 in/sec for the present tests.

Just as in the Erwood tests it was found that once a minimum velocity had been reached which produced 100 percent firing, all higher velocities would also lead to the same result.

Figure 3 shows also that in most cases a slightly higher minimum velocity is required to fire 100 percent of the primers at -40 degrees F than at ambient temperature. The differences are small enough to conclude that when one uses a sufficiently high velocity to be certain of firing at ambient temperature, one also may be certain of firing at -40 degrees F.

Figure 4 gives such safe combinations of firing pin masses and velocities which assure 100 percent firing both at ambient temperature and at -40 degrees F.

VELOCITY OF FIRING PIN (IN/SEC)	16 In-Oz		14 In-Oz		12 In-Oz		10 In-Oz		8 In-Oz		6 In-Oz		5 In-Oz	
	AMB.	$-40^{o}F$	AMB.	$-40^{o}F$	AMB.	$-40^{o}F$	AMB.	$-40^{o}F$	AMB.	$-40^{o}F$	AMB.	$-40^{o}F$	AMB.	$-40^{o}F$
50					23.3	8.0	10.0	4.0	13.3	4.0				
100	100.0	100.0	100.0	85.5	98.9	95.0	86.4	62.0	75.8	61.7	38.8	33.3	53.1	10.0
150	100.0	100.0	100.0	100.0	99.0	94.0	94.8	85.0	88.0	69.3	61.4	45.5	40.6	46.7
200	100.0	100.0	100.0	100.0	100.0	95.8	100.0	91.1	92.8	88.3	82.2	58.0	54.7	45.0
250	100.0	100.0	100.0	100.0	100.0	98.9	100.0	100.0	98.0	92.5	83.3	73.3	70.0	-
300	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	90.0	86.6	-	-
350	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	98.3	98.3	96.6	96.6	-	-
400	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	98.3	99.2	93.3	96.6	-	-
500	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	96.6	93.3	-	-
600	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	-
700	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	96.6
800	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	93.3	-
900	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	90.0	-
1000	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-
1100	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	-
1200	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	90.0

Figure 3
M 42G Primer Firing Test Result
(Percentage of Primers fired at various energy levels, firing pin velocities, and temperatures.)

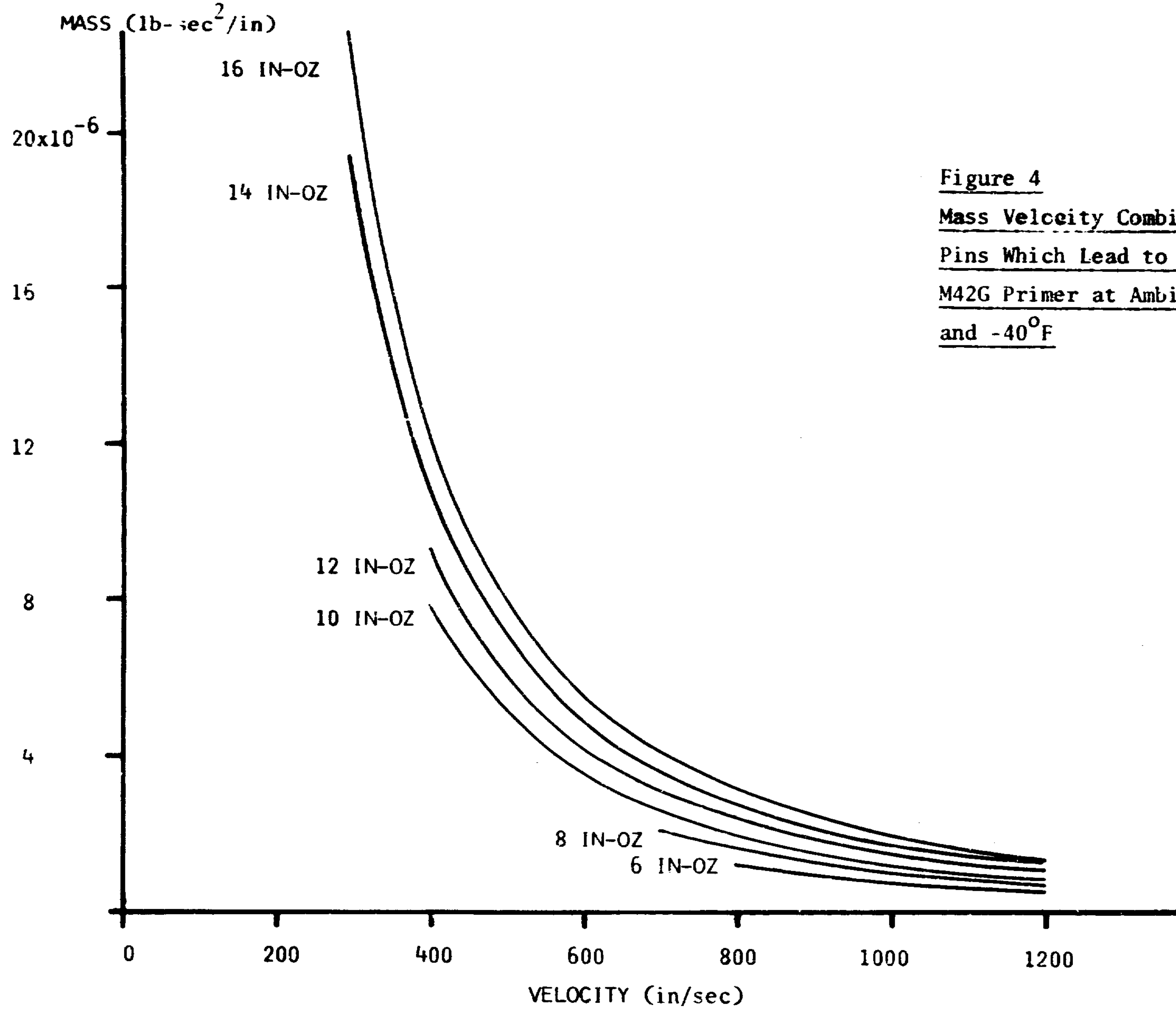


Figure 4

Mass Velccity Combinations for Firing Pins Which Lead to 100% Firing of M42G Primer at Ambient Temperature and -40°F

They were chosen in such a manner that for each energy level the lowest velocity is at least two subdivisions higher than the minimum one which produced 100 percent firing during testing.

Finally, Figures 15a and 15b present the results of the tests in the same form as the Erwood curves.

2. Verification of Theory of Spring Driven Masses

Since the experiment used spring driven firing pins, the data also made it possible to compare the actually attained velocity of each test point, which was held within $\pm$ 2.5 percent of the required nominal velocity v_N, with the theoretical velocity v_T. (See top of page D-28 for discussion on v_N).

As the data sheet of Figure D-13 indicates, the spring compression f was recorded for each test firing, along with the actual mass of the firing pin. Appendix D-4b shows how, together with the spring data of Figure D-9, the above data may be used to compute the theoretical velocity v_T according to Eq. (7).

v_T was found for all test points to be somewhat higher than the corresponding v_N. This is to be expected since the theoretical expression does not make provisions for friction in the system or for certain pecularities of the test setup. (See bottom of page D-33)

For design purposes the above means that to attain a certain nominal velocity v_N, the spring deflection must be such as to produce a somewhat higher theoretical velocity v_T. To this end the design correction factor

$$C_D = \frac{v_T - v_N}{v_N} \tag{10}$$

was introduced. [See Eq. (D-13)]. This permits the following relationship between the theoretical velocity of a given design and the actually desired nominal velocity:

$$v_T = v_N (1 + C_D) \tag{11}$$

[See Eq. (D-12)].

In order to attain a generally applicable value for C_D which can be used in the optimization procedure, its individual values were computed for all tests. Figure D-16 shows this factor to vary between 0.01 and 0.25 for nominal velocities above 100 in/sec. For the sake of safety, it was decided to assume always

$$C_D = 0.25, \tag{12}$$

and accordingly to make Eq. (11)

$$v_T = 1.25\ v_N \tag{13}$$

for the optimization procedure.

IV. Optimization Procedure

Appendix E gives details of the optimization procedure which leads to the optimum design tables presented in Appendix F. The optimization is based on a combination of the theoretical and experimental results of the present investigation. The optimum design tables give complete physical details of spring striker systems with overall lengths of less than 2.000 inches and diameters equal to or less than 0.375 inches. These spring firing pin combinations reproduce the reliable 100 percent firing points given within the heavy outlines of Figure 3.

In addition to reproducing proven firing data with the use of identical masses and velocities as used in the tests, the optimization uses the following criteria:

a. The nominal velocity v_N associated with each test point is to be attained by using Eq. (11) which takes the design correction factor C_D into account.

b. In order to make the springs as short as possible the firing pin velocity v_T is to represent the maximum attainable velocity of the particular system. To this end the spring is to be compressed to its solid height, and the associated corrected shear stress τ_c is to be the maximum allowable one for the material.

c. Only systems with diameters of 0.375 inches are to be initially considered, and only those resulting systems are to be retained which have overall lengths of less than 2.000 inches.

1. Determination of Spring Mass

According to Eq. (E-12) one may write Eq. (7) for the theoretical firing pin velocity v_T with the help of Eq. (11):

$$v_N (1 + C_D) = \frac{\tau_c}{131\ K} \left(- 1 + 2\ e^{-\frac{e^{-\frac{2m_s}{M}}}{2}} \right) \qquad (14)$$

whenever the spring is made of steel.

With v_N and M determined from the test point involved, and with τ_c and C_D given, the required mass of the active turns of the spring is shown by Eq's. (E-13) and (E-14) to be obtained from the above expression:

$$m_s = - \frac{M}{2} \ln [- 2 \ln B] \qquad (15)$$

where

$$B = \frac{131\ K\ v_N\ (1 + C_D)}{2\ \tau_c} + 0.5 \qquad (16)$$

K is the curvature correction factor [42]:

$$K = \frac{4c - 1}{4c - 4} + \frac{0.615}{c} \tag{17}$$

and

$$c = \frac{D}{d} \tag{18}$$

The above shows that the mass of the active turns of the spring depends on the spring index c when M, v_N, τ_c, C_D and D are known.

2. Free Length of Spring and Expanded Length of Spring Corresponding to Maximum Velocity of Firing Pin.

Since the spring is to be compressed through the distance f to its solid height, its free length L_o is determined by:

$$L_o = f + Nd + 2d \tag{19}$$

where N stands for the number of active turns and 2d stands for two full end turns of the spring.

The number of the active turns may be related according to Eq. (E-8) to the mass of the spring in the following way:

$$N = \frac{4\, m_s\, g}{\pi^2 \gamma D d^2} \tag{20}$$

When the above is substituted into Eq. (19) and the deflection f is expressed in terms of the maximum allowable corrected shear stress, one obtains for Eq. (19) according to Eq. (E-10):

$$L_o = \frac{4\, m_s\, g\, c^2}{\pi^2 \gamma\, D^2} \left(\frac{\tau_c\, \pi\, c}{K\, G} + \frac{1}{c} \right) + 2\,\frac{D}{c} \tag{21}$$

where

γ = density of the spring material

G = modulus of rigidity

At the instant when the maximum velocity of the firing pin has been reached, its total trav l is given by Eq. (8). With its help one obtains the expanded length of the spring L_{ex} at this instant from

$$L_{ex} = fC_f + Nd + 2d \tag{22}$$

where C_f is given by Eq. (9). With the appropriate substitutions for f and N, as for Eq. (21), one obtains:

$$L_{ex} = \frac{4\, m_s\, g\, c^2}{\pi^2\, \gamma\, D^2} \left(\frac{\tau_c\, \pi\, c}{K\, G} C_f + \frac{1}{c} \right) + 2 \frac{D}{c}$$

(23)

3. Overall Length of System at Instant of Maximum Firing Pin Velocity

The overall length L_t of the spring- firing pin system at the instant of maximum velocity of the firing pin is given according to Eq. (E-16) by

$$L_t = L_{ex} + L_p \tag{24}$$

where L_p represents the length of the firing pin.

4. Choice of Firing Pin Dimensions and Coil Diameter of Spring

While each successful test point has a definite mass associated with its firing pin, there are many possibilities for the firing pin dimensions. To avoid unnecessary complexity it was decided to assume the use of cylindrical firing pins of constant diameter for the optimization procedure. The .030 in length of the hemispherical tip of the firing pin as well as any possible reduction in diameter, which might accommodate the seating of the pin within the spring, was disregarded.

Figure E-1 shows the results of computations for the length L_p of such steel firing pins for all eligible I.D. numbers, and outside diameters between 0.375 and 0.093 inches. In general, firing pin lengths of less than 0.125 inches and more than 2.000 inches are omitted from the table.

5. Optimization of Overall Length of System

Appendix E indicates that once a choice has been made concerning the outside diameter D_o of the spring and the firing pin, the length L_p of the pin, as well as the spring material, the overall length L_t of the system for a given I.D. number depends only on the spring index c.

While it is not possible to find an analytical expression for the spring index which produces the shortest system, an appropriate computer search which varies the spring index will produce the desired result. The spring index which gives the minimum overall length L_t for a given set of conditions, also furnishes the shortest free length L_o and the shortest expanded length L_{ex} of the spring.

The optimum design tables of Appendix F give the results of such computer searches. Each table represents a single test point (I.D. number) where reliable 100 percent firing occurred, and enumerates the following data for various firing pin outside diameters D_o and associated firing pin length L_p:

a. Mass of active turns of the spring, according to Eq. (15).
b. Wire diameter corresponding to optimum c for given D_o.
c. Number of active turns of the spring, according to Eq. (20).
d. Free length of the spring, corresponding to optimum c, according to Eq. (21).
e. Expanded length of spring, corresponding to optimum c, according to Eq. (23).
f. Overall length of system, corresponding to optimum c, according to Eq. (24).
g. Solid height of spring, including two end turns.

All computations are based on the assumption of steel springs and firing pins, and the following data are used:

γ = 0.283 lbs/in^3

τ_c = 200 000 psi

G = 11.5×10^6 psi

g = 386.05 in/sec^2

C_D = 0.25

[See also discussions in Appendices E and F.]

Figure 5 is typical of the optimum design tables of Appendix F. It treats I.D. No. 67 which had a firing pin velocity v_N of 700 in/sec and a firing pin mass M of 4.0816×10^{-6} lb-sec^2/in at an energy level of 16 inch-ounces.

The following example makes use of one of the designs of I.D. No. 67, and illustrates the use of the tables:

Example

When one chooses an outside diameter D_O = 0.156 in., the following dimensions are given by Figure 5:

Length of firing pin (L_p)	0.291 in.
Mass of active turns of spring (m_s)	3.4957×10^{-6} lb-sec^2/in
Wire diameter (d)	0.03319 in.
Number of active turns of spring (N)	14.3
Number of total turns (N + 2)	16.3
Free length of spring (L_o)	0.786 in.
Expanded length of spring (L_{ex})	0.816 in.
Overall length of system (L_t)	1.109 in
Solid height of spring	0.541 in.

It must now be recalled that the tables give the overall length L_t by considering firing pins where neither the hemispherical tip nor a reduced diameter for seating the pin inside the spring has been accounted for. Let these design factors now be considered. With a spring O.D. of 0.156 in. and a wire size of 0.03319 in. the inside diameter of the spring is approximately 0.089 in. Allowing for sufficient clearance, one lets the seating diameter of the firing pin be 0.080 in. and its length 0.125 in. Figure 6a shows the resulting firing pin design which maintains the mass of the pin at 4.0816×10^{-6}lb-sec^2/in, while allowing for the addition of the seating diameter as well as the hemispherical tip. The resulting length of the 0.156 diameter is 0.255 in., and together with the 0.030 length of the tip the active length of the firing pin becomes 0.285 in. This length must be added to the solid height of the spring to determine the nominal space which the system requires when the spring is compressed. (It serves to locate the release mechanism.) This length must also be added to the expanded spring length to determine the space requirement at maximum velocity firing. Figure 6b shows the first dimension as 0.541 + 0.285 = 0.826 in., (not taking some necessary clearances into account). The actual overall length of the system has to be 0.816 + 0.285 = 1.101 in.

IN-OZ = 16 I.D. NO. = 67
VELOCITY = 700.0 PIN MASS = 4.081

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.218	.148	3.3964	.04542	5.3	0.463	0.478	0.626	.330
.200	.177	3.3964	.04167	6.8	0.525	0.542	0.721	.368
.187	.202	3.4957	.03979	8.3	0.580	0.602	0.805	.410
.170	.245	3.4957	.03617	11.0	0.678	0.704	0.950	.472
.156	.291	3.4957	.03319	14.3	0.786	0.816	1.109	.541
.140	.362	3.4957	.02979	19.8	0.953	0.990	1.355	.648
.125	.454	3.4957	.02660	27.8	1.174	1.220	1.678	.792

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

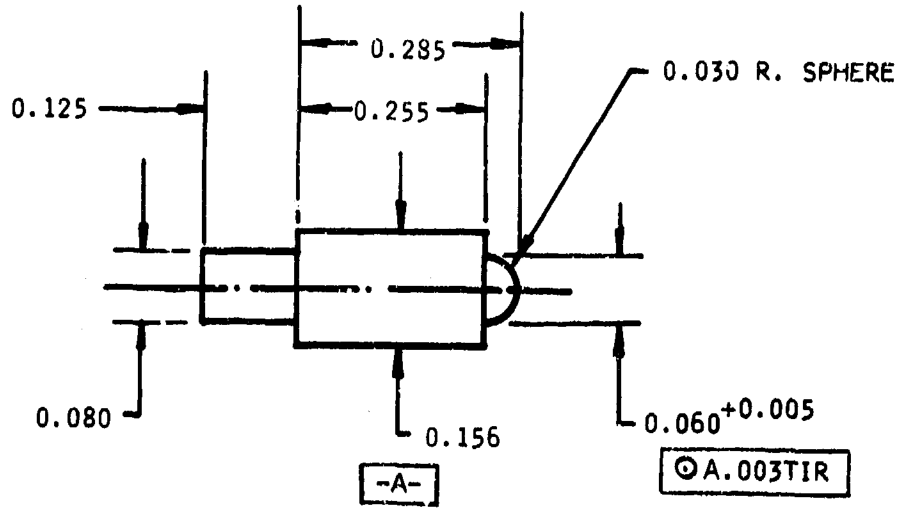
ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

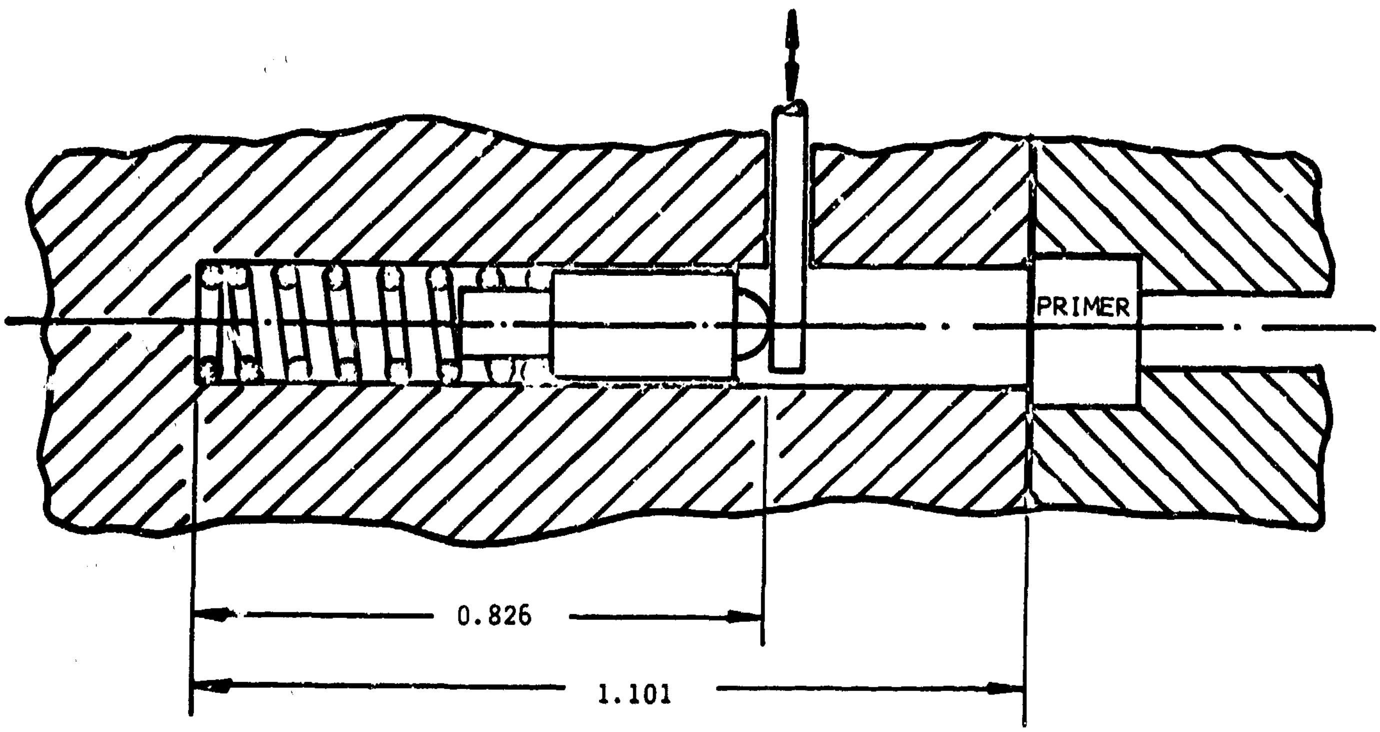
ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

Figure 5
Typical Optimum Design Table



a.



b.

<u>Figure 6</u>

<u>Typical Application of Optimum Design Tables</u>

a. Firing pin design

b. Space requirements for system both when spring is compressed and when expanded

Minor size deviations from the optimum design tables due to necessary clearances and part tolerances, (including changes in wire size of 0.0003 in.), will not influence the results to any extent since the factor C_D has been chosen sufficiently high.

It has to be kept in mind though that in order to avoid spring set during assembly the corrected shear stress τc should never exceed its upper limit of approximately 200,000 psi at solid height.

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APPENDIX A

HELICAL SPRING AS A DISTRIBUTED SYSTEM

1. Derivation of Partial Differential Equation

a. Definition of Terms

Figure A-1 shows a helical spring which is held on one end against the fixed surface S while its other end is deflected through the distance f by the axially applied force P_f. The indicated x-coordinate is measured along the helically shaped centerline of the spring wire. The y-axis is coincident with the coil axis, and the coordinate $y = y(x,t)$ describes the instantaneous positions of all points along the x-axis.

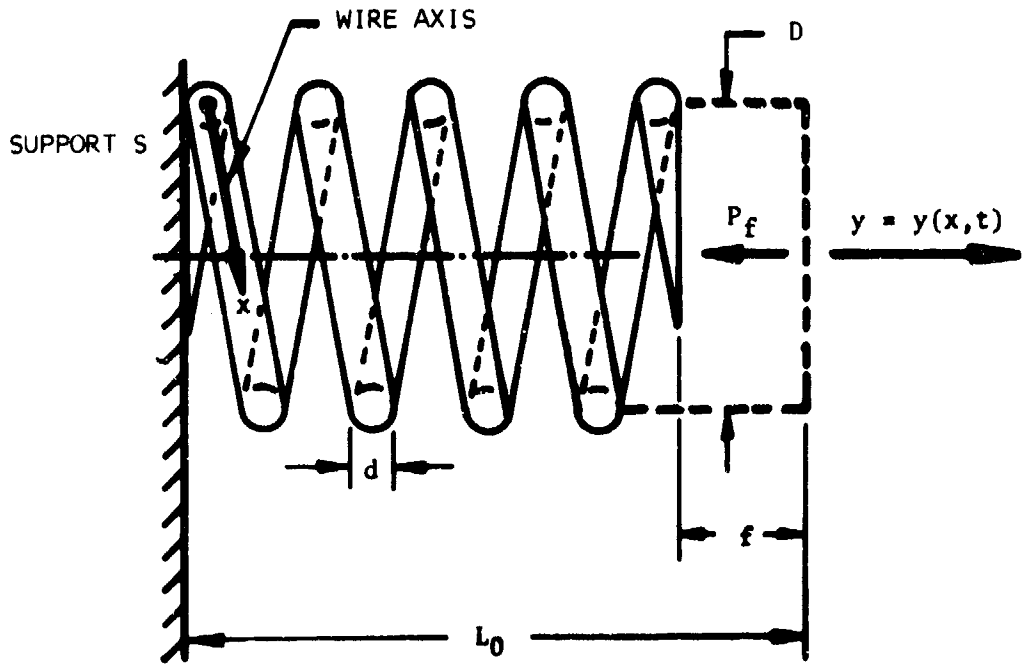


Figure A-1

Helical Spring Terminology

The following terms must now be defined:

d = wire diameter
D = mean coil diameter
L = length of spring wire (along x-axis and embracing active turns)
N = number of active turns
L_0 = free length of spring
f' = f/N, deflection per coil due to force P_f
A = cross-sectional area of wire ($\pi d^2/4$)
J_0 = polar moment of inertia of wire cross-section ($\pi d^4/32$)
γ = density of spring material
g = 386.05, acceleration of gravity (in/sec^2)
G = modulus of rigidity

The spring constant (λ) of a helical spring is given by:

$$\lambda = \frac{P_f}{f} = \frac{Gd^4}{8D^3N} \tag{A-1a}$$

The spring constant per coil of the spring becomes:

$$c' = \frac{P_f}{f'} = \lambda N = \frac{Gd^4}{8D^3} \tag{A-1b}$$

The spring constant per unit length of spring wire is given by:

$$k = \lambda L = \frac{Gd^4 L}{8D^3N} = \frac{Gd^4(\pi DN)}{8D^3N} = \frac{Gd^4\pi}{8D^2} \tag{A-1c}$$

b. External Forces Acting on Element of Length dx

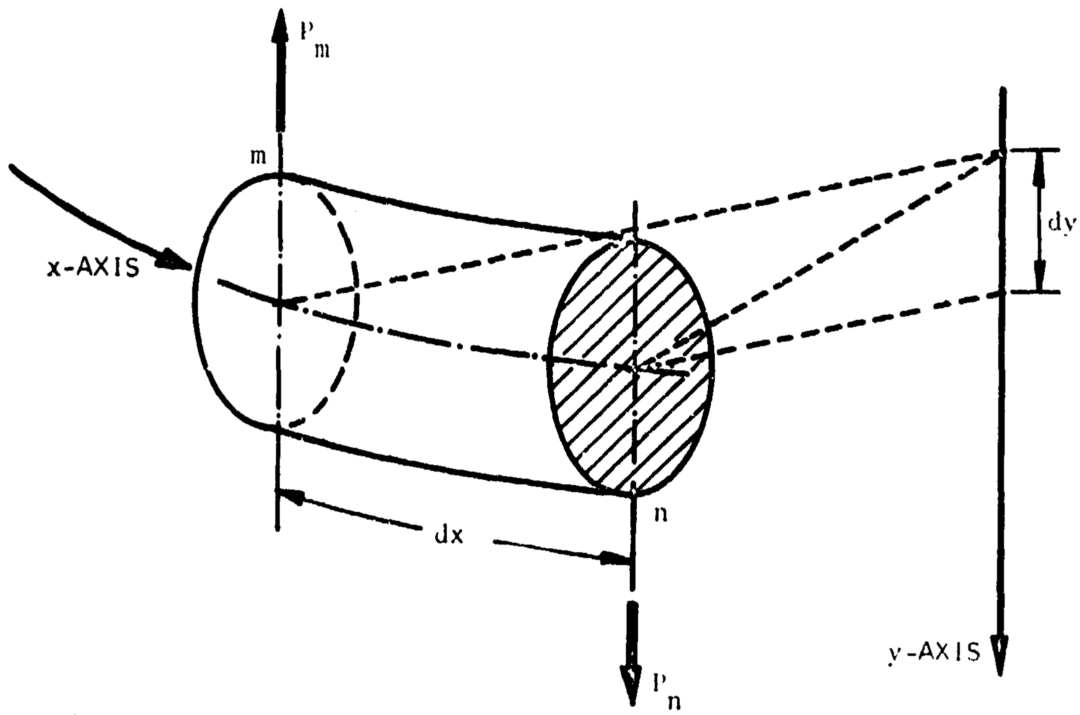


Figure A-2

Spring Element of Length dx

If the element of length dx of Figure A-2 is deflected so that the axial displacement corresponds to dy, one may use the following proportionality

$$\frac{dx}{L/N} = \frac{dy}{f'}, \tag{A-2}$$

i.e. dx bears the same relationship to the length of a single coil as dy has to the deflection of a single coil due to an applied force. Therefore using partial derivatives

$$\frac{\partial y}{\partial x} = \frac{f'N}{L} ; \qquad \text{(A-3)}$$

now consider the force in the y-direction at section m according to Eq.(A-1b)

$$P_m = c'f' \qquad \text{(A-4)}$$

and substituting the value of f' according to Eq.(A-3) gives:

$$P_m = \frac{c'L}{N} \frac{\partial y}{\partial x} \qquad \text{(A-5)}$$

The force at section n is obtained by a Taylor Series expansion of P_m, i.e.

$$P_n = P_m + \frac{\partial}{\partial x}(P_m)dx = P_m + \frac{c'L}{N} \frac{\partial^2 y}{\partial x^2} dx \qquad \text{(A-6)}$$

Assuming that P_n acts in the direction of the positive y-axis. the sum of the external forces ΣF, acting on the element is given by:

$$\Sigma F = P_n - P_m = P_m - \frac{c'L}{N} \frac{\partial^2 y}{\partial x^2} dx - P_m$$

or

$$\Sigma F = \frac{c'L}{N} \frac{\partial^2 y}{\partial x^2} dx \qquad \text{(A-7)}$$

c. D'Alembert Forces Acting on Element of Length dx

Translational D'Alembert Force

The D'Alembert force due to the translational acceleration of the element of mass dm is given by:

$$F_{Dt} = - dm \frac{\partial^2 y}{\partial t^2} \qquad \text{(A-8a)}$$

With

$$dm = \frac{\gamma \pi d^2}{4g} dx$$

the above becomes:

$$F_{Dt} = - \frac{\gamma \pi d^2}{4g} dx \frac{\partial^2 y}{\partial t^2} \qquad \text{(A-8b)}$$

D'Alembert Force Derivable from D'Alembert Moment

Figure A-3 illustrates that the application of an axial force in the direction of the positive y-axis does not only lead to the axial deflection dy of the wire, but also causes it to rotate about the x-axis through a counterclockwise angle $d\phi$.

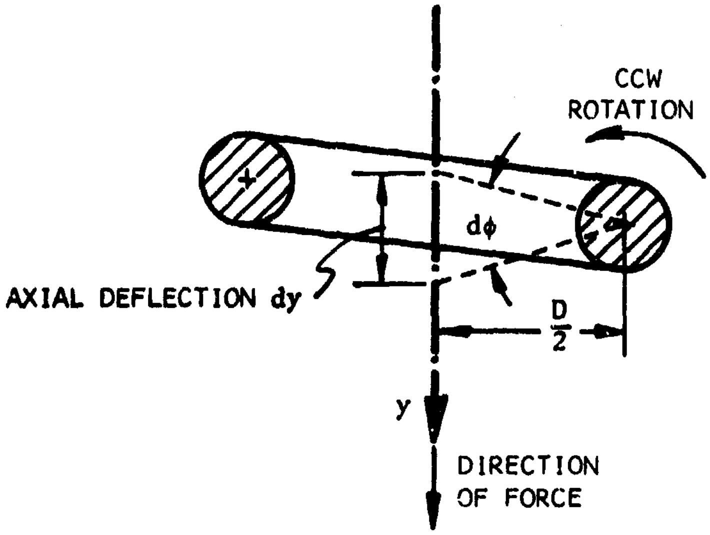


Figure A-3

Rotation About Wire Axis Due to Axial Deflection

This rotation leads to the angular acceleration $d^2\phi/dt^2$, and therefore to the following D'Alembert moment per unit length dx:

$$M_D = - \frac{J_o \gamma}{g} dx \frac{d^2\phi}{dt^2} \qquad \text{(A-9)}$$

Since any moment may be expressed in terms of the product of a force and a distance, one introduces a "rotational D'Alembert force" (T_{Dr}) which is defined by:

$$M_D = T_{Dr} \frac{D}{2}$$

where D represents the mean diameter of the coil. Then

$$T_{Dr} = M_D \frac{2}{D}$$

Equation (A-9) then becomes:

$$T_{Dr} = - \frac{2J_0\gamma}{gD} dx \frac{d^2\phi}{dt^2} \tag{A-10}$$

The angle $d\phi$ is related to the axial deflection dy by:

$$dy = 2 \frac{D}{2} \sin\frac{d\phi}{2} \approx \frac{D}{2} d\phi \tag{A-11}$$

Using partial derivatives one obtains:

$$\frac{\partial^2\phi}{\partial t^2} = \frac{2}{D} \frac{\partial^2 y}{\partial t^2} \tag{A-12}$$

Substitution of Eq.(A-12) into Eq.(A-10) results in:

$$T_{Dr} = - \frac{4J_0\gamma}{gD^2} dx \frac{\partial^2 y}{\partial t^2} \tag{A-13}$$

d. <u>Partial Differential Equation of Motion</u>

The differential equation of motion of the helical spring with circular wire is obtained with the help of D'Alembert's Principle·

$$\Sigma F + \Sigma T_D = 0 \tag{A-14}$$

Substitution of Eq's.(A-7), (A-8b) and (A-13) into the above yields:

$$\frac{c'L}{N} \frac{\partial^2 y dx}{\partial x^2} - \frac{\gamma}{g}\left[\frac{\pi d^2}{4} + \frac{4J_0}{D^2}\right] \frac{\partial^2 y}{\partial t^2} dx = 0 \tag{A-15}$$

Rearrangement leads to the following result:

$$a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2} \tag{A-16}$$

where

$$a^2 = \frac{c' Lg}{N\gamma \left[\frac{\pi d^2}{4} + \frac{4J_0}{D^2}\right]} = \frac{\lambda Lg}{\gamma \left[\frac{\pi d^2}{4} + \frac{4J_0}{D^2}\right]} \qquad \text{(A-17)}$$

e. Influence of Term Containing Polar Moment of Inertia J_0 on the Velocity of Wave Propagation

The expression

$$a = \sqrt{\frac{\lambda Lg}{\gamma \left[\frac{\pi d^2}{4} + \frac{4J_0}{D^2}\right]}} \qquad \text{(A-18)}$$

as given by Eq.(A-17) represents the velocity of wave propagation or surge velocity of a helical spring with circular wire. (See also Appendix B-3b.) It represents the velocity with which a disturbance is propagated along the x-coordinate of the spring. In order to examine the influence of the term $4J/D^2$, which accounts for the rotational inertia of the spring wire, let Eq.(A-18) be written in the following form:

$$a = \sqrt{\frac{1}{1 + 0.5\left(\frac{d}{D}\right)^2}} \sqrt{\frac{\lambda Lg}{\gamma \left(\frac{\pi d^2}{4}\right)}} \qquad \text{(A-18)}$$

The term $\sqrt{\frac{1}{1 + 0.5(d/D)^2}}$ equals 0.990 for a spring index D/d = 5, and it becomes equal to 0.994 when D/d = 10. Since the above range of spring indices is usually encountered in practice, one may write Eq.(A-18) for all springs in the following form:

$$a = 0.99 \sqrt{\frac{\lambda Lg}{\gamma \left(\frac{\pi d^2}{4}\right)}} \qquad \text{(A-19)}$$

2. Determination of Maximum Attainable Velocity at Free End of Helical Spring

The following uses the partial differential equation (A-16) to determine the displacement $y(x,t)$ as well as the velocity $\dot{y}(x,t)$ of a helical spring which is fixed on one end to the support S (see Figure A-1), and whose other end is initially compressed a distance (-)f before it is released.

The solution of the above makes it possible to find the maximum attainable velocity of the free end of the spring.

a. Boundary Conditions

Since the spring wire remains fixed to the support(s) at all times

$$y(0,t) = 0 \tag{A-20a}$$

The spring is free at $x = L$. This is expressed by saying that there is no force at this point and therefore there is no strain. The force at any element was described by Eq.(A-5)

$$P_m = \frac{c'L}{N}\frac{\partial y}{\partial x}$$

Since $c'L/N \neq 0$, for $P_{m(x = L)} = 0$, the second boundary condition must be:

$$\frac{\partial y(L,t)}{\partial x} = 0 \tag{A-20b}$$

b. Initial Conditions

Let it be assumed that the initial deflection of all elements along the x-axis is proportional to the deflection (-f) at $x = L$. Thus

$$y(x,0) = -\frac{f}{L}x = f(x) \tag{A-21a}$$

Since all parts of the spring are standing still when $t = 0$, the second initial condition becomes:

$$\frac{\partial y(x,0)}{\partial t} = 0 = g(x) \tag{A-21b}$$

c. Solution of the Equation of Motion

Using the method of separation of variables for the solution, one lets

$$y(x,t) = X(x)T(t) \tag{A-22}$$

To satisfy Eq.(A-16), one must let

$$X(x) = C_1\cos\lambda x + C_2\sin\lambda x \tag{A-23}$$

while

$$T(t) = C_3\cos\lambda at + C_4\sin\lambda at \qquad \text{(A-24)}$$

The coefficients C_1 and C_2 are now found from the boundary conditions of Eq's. (A-21a) and (A-21b)

$$X(0) = 0 = C_1(1) + C_2(0) \qquad \text{(A-25a)}$$

and thus

$$C_1 = 0 \qquad \text{(A-26)}$$

Furthermore

$$X'(L) = 0 = C_2\lambda\cos\lambda L \qquad \text{(A-25b)}$$

For a nontrivial solution, where $C_2 \neq 0$, one must satisfy

$$\cos\lambda L = 0 \ .$$

This occurs when

$$\lambda L = \frac{n\pi}{2} \qquad (n=1,3,5,\ldots \text{ odd}) \ ,$$

and thus the eigenvalue λ must be

$$\lambda = \frac{n\pi}{2L} \qquad \text{(A-27)}$$

Equation (A-23) then becomes:

$$X_n(x) = C_n\sin\frac{n\pi}{2L}x \qquad \text{(A-28)}$$

i.e. there will be an $X_n(x)$ for each n.
Similarly Eq.(A-24) takes the form:

$$T_n(t) = A_n\cos\frac{n\pi a}{2L}t + B_n\sin\frac{n\pi a}{2L}t \qquad \text{(A-29)}$$

The total solution is a summation of all $X_n(x)T_n(t)$ according to Eq.(A-22):

$$y(x,t) = \sum_{n=1,3,\ldots \text{ odd}}^{\infty} \left(D_n\cos\frac{n\pi a}{2L}t + E_n\frac{n\pi a}{2L}t\right)\sin\frac{n\pi}{2L}x \qquad \text{(A-30)}$$

Since Eq.(A-21b) shows that all initial velocities are zero, i.e. $g(x) = 0$, it follows that

$$E_n = 0 \qquad \text{(A-31a)}$$

Equation (A-21a) serves for the evaluation of D_n. Thus:

$$y(x,0) = f(x) = -\frac{f}{L}x = \sum_{1,3,\ldots \text{ odd}}^{n} D_n \sin\frac{n\pi}{2L}x \qquad \text{(A-31b)}$$

To evaluate D_n multiply both sides of Eq.(A-31b) by $\sin\frac{m\pi}{2L}x\, dx$ and integrate between 0 and L:

$$-\frac{f}{L}\int_0^x x \sin\frac{m\pi}{2L}x\, dx = \sum_{1,3,\ldots \text{ odd}}^{n} D_n \int_0^L \sin\frac{n\pi}{2L}x \sin\frac{m\pi}{2L}x\, dx \qquad \text{(A-32)}$$

For $m \neq n$ the integral on the right hand side of Eq.(A-32) vanishes, i.e.

$$\int_0^L \sin\frac{n\pi}{2L}x \sin\frac{m\pi}{2L}x\, dx = \frac{1}{2}\int_0^L \cos(n-m)\frac{\pi}{2L}x\, dx - \frac{1}{2}\int_0^L \cos(n+m)\frac{\pi}{2L}x\, dx$$

$$= \frac{L}{\pi}\left[\frac{\sin(n-m)}{(n-m)}\frac{\pi x}{2L}\Bigg|_0^L - \frac{\sin(n+m)}{(n+m)}\frac{\pi x}{2L}\Bigg|_0^L\right] = 0$$

The above is proven as follows:

a. For m = odd: Since n = odd, all (n-m) as well as (n+m) will be even, $\sin(n-m)\pi/2$ and $\sin(n+m)\pi/2$ will have the form $\sin(k\pi)$, where k = 1,2,3,....

b. For m = even: Since n = odd, all (n-m) and (n+m) will be odd, and the difference between (n+m) and (n-m) will be equal to 2m. Thus $(n+m)\pi/2$ and $(n-m)\pi/2$ are angles which are $m\pi$ radians out of phase, and the signs and values of the sine functions are identical.

For m = n Eq.(A-32) becomes:

$$-\frac{f}{L}\int_0^L x \sin\frac{n\pi}{2L}x\, dx = \sum_{1,3,\ldots \text{ odd}}^{n} D_n \int_0^L \sin^2\frac{n\pi}{2L}x\, dx \qquad \text{(A-33)}$$

Since

$$\int_0^L \sin^2\frac{n\pi}{2L}x\, dx = \frac{1}{2}\int_0^L \left(1 - \cos\frac{n\pi}{L}x\right) dx = \frac{L}{2}, \qquad \text{(A-34)}$$

one obtains

$$D_n = -\frac{2f}{L^2}\int_0^L x \sin\frac{n\pi}{2L}x \, dx \tag{A-35}$$

Integration by parts of the above leads to:

$$D_n = -\frac{8f}{n^2\pi^2}\sin\frac{n\pi}{2} \tag{A-36}$$

Equations (A-31a) and (A-36) are now substituted into Eq.(A-30), and the solution is given by:

$$y(x,t) = \sum_{n=1,3,\ldots \text{ odd}}^{\infty} -\frac{8f}{n^2\pi^2}\sin\frac{n\pi}{2}\sin\frac{n\pi}{2L}x\cos\frac{n\pi}{2L}at \tag{A-37}$$

d. Maximum Velocity of Free End of Spring

Equation (A-37) becomes for the free end of the spring, i.e. for $x = L$:

$$y(L,t) = \sum_{n=1,3,\ldots \text{ odd}}^{\infty} -\frac{8f}{n^2\pi^2}\sin^2\frac{n\pi}{2}\cos\frac{n\pi}{2L}at \tag{A-38a}$$

and since

$$\sin^2\frac{n\pi}{2} = 1 \,, \quad \text{for } n=1,3,\ldots \text{ odd}$$

one may write the above as

$$y(L,t) = \sum_{n=1,3,\ldots \text{ odd}}^{\infty} -\frac{8f}{n^2\pi^2}\cos\frac{n\pi}{2L}at \tag{A-38b}$$

The velocity at $x = L$ is given by differentiation of the above:

$$\frac{\partial y}{\partial t}(L,t) = \sum_{n=1,3,\ldots \text{ odd}}^{\infty} \frac{4fa}{n\pi L}\sin\frac{n\pi}{2L}at \tag{A-39}$$

To find the maximum attainable velocity at $x = L$, one differentiates again with respect to time and sets the result equal to zero:

$$\frac{\partial^2 y(L,t)}{\partial t^2} = 0 = \sum_{n=1,3,\ldots \text{ odd}}^{\infty} \frac{4fa^2}{2L^2}\cos\frac{n\pi}{2L}at \tag{A-40}$$

The series will vanish for all n for the first time when

$$\frac{at}{L} = 1$$

i.e. when,

$$t = T_S = \frac{L}{a} \tag{A-41}$$

$t = T_S$ is known as the surge time. This is the time it takes for the releasing tension wave to travel along the full length (L) of the wire with the surge velocity (a). (See Appendix B.)

Substitution of Eq.(A-41) into Eq.(A-39) gives the maximum velocity at x = L during the first cyclic motion of the spring:

$$V_{MAX_L} = \frac{\partial y}{\partial t}(L,\frac{L}{a}) = \sum_{n=1,3,\ldots \text{ odd}}^{\infty} \frac{4fa}{n\pi L} \sin\frac{n\pi}{2} \tag{A-42}$$

To evaluate the above consider $\frac{1}{n}\sin\frac{(n\pi)}{2}$ for various values of n = odd:

n = 1: $\frac{1}{n}\sin\frac{(n\pi)}{2} = 1$

n = 3: $\frac{1}{n}\sin\frac{(n\pi)}{2} = -\frac{1}{3}$

n = 5: $\frac{1}{n}\sin\frac{(n\pi)}{2} = \frac{1}{5}$; etc.

Since

$$\sum_{n=1,3,\ldots \text{ odd}}^{\infty} \frac{\sin n\pi/2}{n} = \frac{\pi}{4}$$

equation (A-42) may be expressed as

$$V_{MAX_L} = \frac{fa}{L} \tag{A-43}$$

Now if Eq.(A-19) is substituted for (a) in the above, one obtains:

$$V_{MAX_L} = V_H = 0.99f\sqrt{\frac{\lambda g}{L\gamma\ (\frac{\pi d^2}{4})}} \tag{A-44}$$

The symbol V_H is used throughout this report for the maximum velocity attainable by the free end of a helical spring (which does <u>not</u> drive any mass).

Now consider that the deflection f may be expressed in terms of the spring constant λ, and the respective load P_f, i.e.

$$f = \frac{P_f}{\lambda}$$

Further, the shear stress τ_f which corresponds to f is given by

$$\tau_f = \frac{8P_f D}{\pi d^3} \qquad \text{(A-45)}$$

so that

$$f = \frac{\tau_f \pi d^3}{8D\lambda} \qquad \text{(A-46)}$$

Substitution of the above into Eq.(A-44) leads to

$$V_H = 0.99\tau_f\sqrt{\frac{\pi d^4 g}{16\gamma D^2 L\lambda}} \qquad \text{(A-47)}$$

Further substitution of Eq.(A-1c) for the spring constant per unit length of wire (λL) gives

$$V_H = 0.99\tau_f\sqrt{\frac{g}{2\gamma G}} \qquad \text{(A-48)}$$

NOTE: For a spring made of steel with $\gamma = 0.283$ lb/in^3 and $G = 11.5 \times 10^6$

$$V_H = \frac{\tau_f}{131} \ \text{(in/sec)} \qquad \text{(A-49)}$$

With a permissible $\tau_f = 200{,}000$ psi, (in absence of curvature correction), the velocity of the free end of a spring becomes 1527 in/sec.

APPENDIX B

DETERMINATION OF VELOCITIES OF FREELY EXPANDING SPRINGS AND SPRING DRIVEN MASSES BY MEANS OF WAVE PROPAGATION THEORY

K. Maier [35-37] has shown without proof that wave propagation theory may be used to excellent advantage for the determination of spring velocities under various circumstances. This approach furnishes simple design equations without requiring the knowledge of eigenvalues of the partial differential equation. (See Appendix A.)

The following work gives the derivations of the above mentioned design equations, starting with the underlying principles of wave propagation in prismatical bars [31].

1. Longitudinal Pressure Waves in Thin Prismatical Bars

Concepts of Particle Velocity and Velocity of Wave Propagation

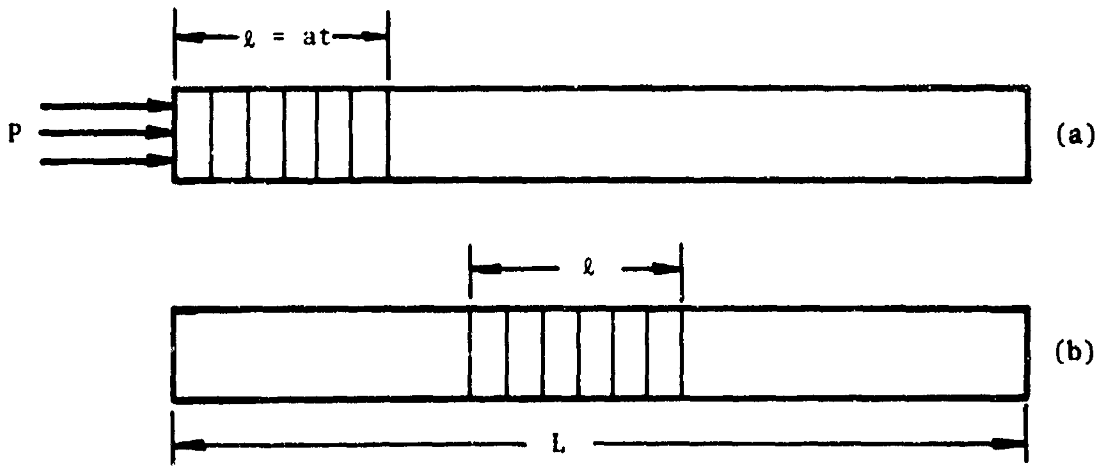


Figure B-1

Initiation and Transmission of Pressure Wave

a. Pressure wave of length ℓ is initiated.
b. Pressure wave travels across bar.

Figure B-1 shows a thin long uniform bar of elastic material. If a uniformly distributed load P is suddenly applied to the end of the bar, and acts for a time t, then a pressure wave will be started at this end, and it will move along the bar. This pressure wave represents a zone in which the material of the bar is under compression.

In front of the zone as well as behind it the material is entirely uncompressed and at rest. At each instant new material in front of the zone is about to be compressed, while at the same time material at the rear of the zone is losing its compression.

Thus the zone can move along the full length of the bar, although individual particles have only the small motion involved in compression and decompression as the zone passes.

The velocity with which the pressure zone advances along the bar is termed the wave propagation velocity (a), while the velocity of the individual particles within the pressure zone is called the particle velocity (v). This phenomenon may be explained as follows: When the force P is applied it compresses the material near the end of the bar. The compressed portion is shortened slightly, i.e. some of it experiences a displacement, and in being displaced is given a velocity. (This velocity is towards the right in Figure B-1.) It strikes the stationary material in front of it and compresses it, and starts it moving. In so doing it is stopped itself, and the zone of compression moves foward. Thus the material in the compressed zone always has a certain velocity, while the material in front of it and behind it is at rest.

Expressions for the particle velocity as well as for the velocity of wave propagation will now be derived. Let the following magnitudes be defined:

P	=	magnitude of compressive force
t	=	time during which the compressive force acts
L	=	length of bar
ℓ	=	length of compressive or tensile zone
v	=	particle velocity
a	=	velocity of wave propagation
E	=	modulus of elasticity
A	=	cross-sectional area of the bar
m	=	$\frac{\gamma A(1)}{g}$, the mass per unit length of bar
γ	=	ρg, density of bar material (lb/in^3)
g	=	gravitational constant
k	=	EA, stiffness constant per unit length of bar, from $E = \frac{PL}{A\,dL}$ one obtains the stiffness of the total bar as $\frac{P}{dL} = \frac{EA}{L}$. When L equals unity, the above expression results.

a. Particle Velocity

When the zone of length ℓ, (see Figure B-1), is compressed by the force P, it is foreshortened by the amount

$$d\ell = \frac{P\ell}{EA} = \frac{P\ell}{k} \tag{B-1}$$

During this period force P does the work

$$W_p = P\,d\ell = \frac{P^2\ell}{k} \tag{B-2}$$

The kinetic energy of the mass in the zone of length ℓ is given by:

$$E_K = \frac{1}{2}m\ell v^2 \tag{B-3}$$

The strain energy of this zone is found from the unit strain energy in compression or tension:

$$E_{su} = \frac{1}{2}\sigma\,\varepsilon \qquad \text{(B-4)}$$

where

σ = compressive or tensile stress

ε = compressive or tensile strain due to σ

Thus for the zone of volume $A\ell$ one obtains

$$E_s = \frac{1}{2}\frac{P}{A}\frac{d\ell}{\ell}(\ell A) = \frac{P}{2A}\frac{P}{k}(\ell A) = \frac{P^2\ell}{2k} \qquad \text{(B-5)}$$

The energy balance of the compressed zone is given by:

$$W_P = E_K + E_s \qquad \text{(B-6)}$$

or with the help of Eq's.(B-3), (B-4) and (B-5)

$$\frac{P^2\ell}{k} = \frac{m\ell v^2}{2} + \frac{P^2\ell}{2k} \qquad \text{(B-7)}$$

Note that the above shows that the total energy in the zone is half potential and half kinetic. Equation (B-7) is now used to determine the particle velocity:

$$v = P\sqrt{\frac{1}{mk}} \qquad \text{(B-8)}$$

When the above is expressed in terms of the stress and the mass density ρ, one obtains:

$$v = \sigma\sqrt{\frac{1}{E\rho}} \qquad \text{(B-9)}$$

Equation (B-9) shows that the particle velocity depends entirely on the stress applied to a given bar.

b. Velocity of Wave Propagation

During a time dt the pressure zone advances the distance adt, where a is the velocity of wave propagation.

The associated change of momentum experienced by an element of length adt is given by:

$$m\,(adt)\,(v - 0) = m\,(adt)\,P\sqrt{\frac{1}{mk}} \qquad \text{(B-10)}$$

According to the Principle of Linear Momentum, this momentum change equals the impulse experienced by the mass involved. Since the compression P represents the only force acting, one obtains:

$$P\,dt = m\,(adt)\,P\sqrt{\frac{1}{mk}} \qquad \text{(B-11)}$$

The velocity of wave propagation may now be determined as

$$a = \sqrt{\frac{k}{m}} \tag{B-12}$$

or in different terms:

$$a = \sqrt{\frac{E}{\rho}} \tag{B-13}$$

Equation (B-13) indicates that the wave propagation velocity depends entirely on the material properties of the bar.

It should still be pointed out that when dealing with the above described compression wave, the velocity of the individual particles has the same direction as that of the advancing wave. In a tension wave the velocity of the individual particles is in the opposite direction to that of the wave propagation.

2. Superposition of Waves

Wave Reflections at Free and Fixed Ends of Bars

Superposition of waves is valid as long as the material follows Hooke's Law, and there is no friction. The resulting force or stress level, as well as the resulting particle velocity are the vectorial sums of the respective components. In the following discussion it is assumed that the bar is of constant cross-section and of the same material throughout.

a. Two Compression (or Tension) Waves Meet

Figure B-2 depicts the conditions of the forces and the particle velocities when two compression waves meet. It is assumed that both ends of a prismatic bar experience suddenly applied and uniformly distributed compression loads of different magnitudes and durations. Similar conditions would prevail if both waves were produced by tensile loads. Part (a) shows the two waves of compression $\overline{P}_1$ and $\overline{P}_2$ traveling towards each other with identical absolute values of wave propagation velocity a. The particle velocities of both waves have the same directions as their associated wave propagation velocities. Since $\overline{v}_1$ is defined as having a positive direction, the direction of $\overline{v}_2$ is counted as negative. (In the case of two tension waves these signs would be reversed.)

When the waves meet, as shown in part (b), the resulting compression force $\overline{P}_R$ becomes:

$$\overline{P}_R = \overline{P}_1 + \overline{P}_2 = \overline{P_1 + P_2} \tag{B-14}$$

The resulting particle velocity is obtained with the help of Eq.(B-8):

$$\overline{v}_R = \overline{v}_1 + \overline{v}_2 = \overline{v_1 - v_2} = \frac{\overline{P_1 - P_2}}{\sqrt{mk}} \tag{B-15}$$

Thus the stress level is increased, while the particle velocity becomes smaller than the largest component velocity.

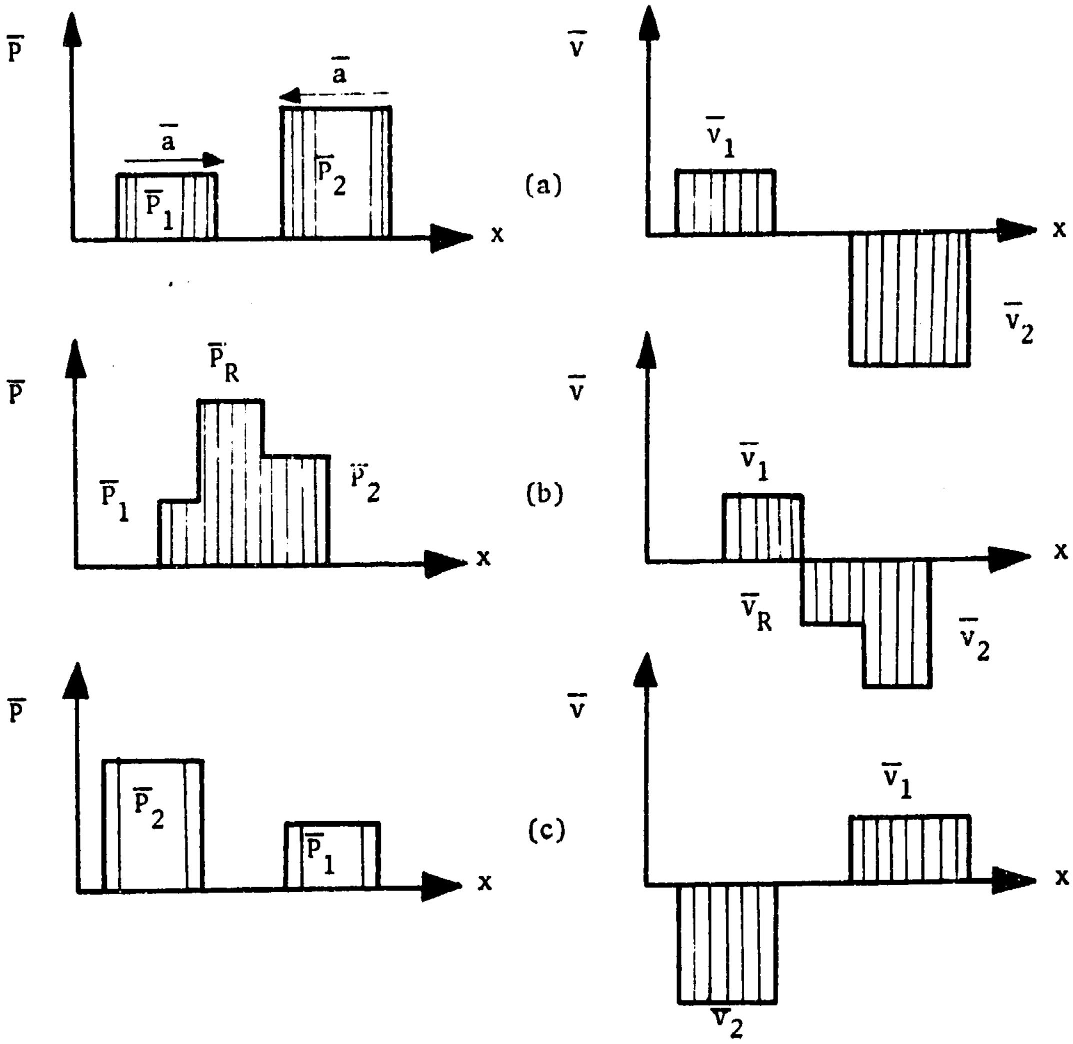


Figure B-2

Two Compression Waves Meet

a. Before meeting

b. At meeting

c. After meeting

Part(c) shows that once the waves have passed each other, each continues with its original stress and particle velocity.

The above allows a certain deduction concerning the stress at the fixed end of a bar or a beam. Due to the fixity there cannot be any particle velocity at such a point. To assure this condition, the stress of the reflected wave must be of the same magnitude and sign as that of the incident wave. This causes the particle velocities of the reflected and incident waves to be equal in magnitude but of opposite signs. While the particle velocity at such an end point becomes zero, the stress doubles.

b. One Compression and One Tension Wave Meet

Figure B-3 indicates the conditions of the forces and the particle velocities when a compression and a tension wave meet. Part (a) shows both waves traveling towards each other with identical absolute values of wave propagation velocity a. Load $\overline{P}_1$ is compressive, while load $\overline{P}_2$ is tensile. The particle velocities $\overline{v}_1$ and $\overline{v}_2$ have identical signs and are positive.

Part (b) shows the meeting of these waves. Because of the signs of the component forces one finds the resultant force $\overline{P}_R$ smaller than either of the component forces, i.e.:

$$\overline{P}_R = \overline{P}_1 + \overline{P}_2 = \overline{P_1 - P_2} \tag{B-16}$$

On the other hand, the resulting particle velocity $\overline{v}_R$ is larger than that of either of the component waves:

$$v_R = \overline{v}_1 + \overline{v}_2 = \overline{v_1 + v_2} = \frac{\overline{P_1 + P_2}}{\sqrt{mk}} \tag{B-17}$$

Part (c) again indicates that once the waves have passed each other, each continues with its original properties.

The presently discussed case allows a deduction concerning the particle velocity at the free end of a bar or beam. Since there will be no opposing forces, the stress at such a free end must be zero. To assure this condition, the stress of the reflected wave must be of the same magnitude, but of opposite sign, as that of the incident wave. As a consequence the particle velocity of the reflected wave is equal to that of the incident wave both in magnitude as well as in sign. While the stress becomes zero in this manner, the particle velocity doubles at the free end.

Up to now only waves produced by constant forces were considered. The stress σ and the particle velocity v were constant along the wave. In the case of a variable force, a wave will be produced in which σ and v vary along the length. Conclusions obtained above regarding propagation, superposition and reflection of waves can also be applied in this more general case.

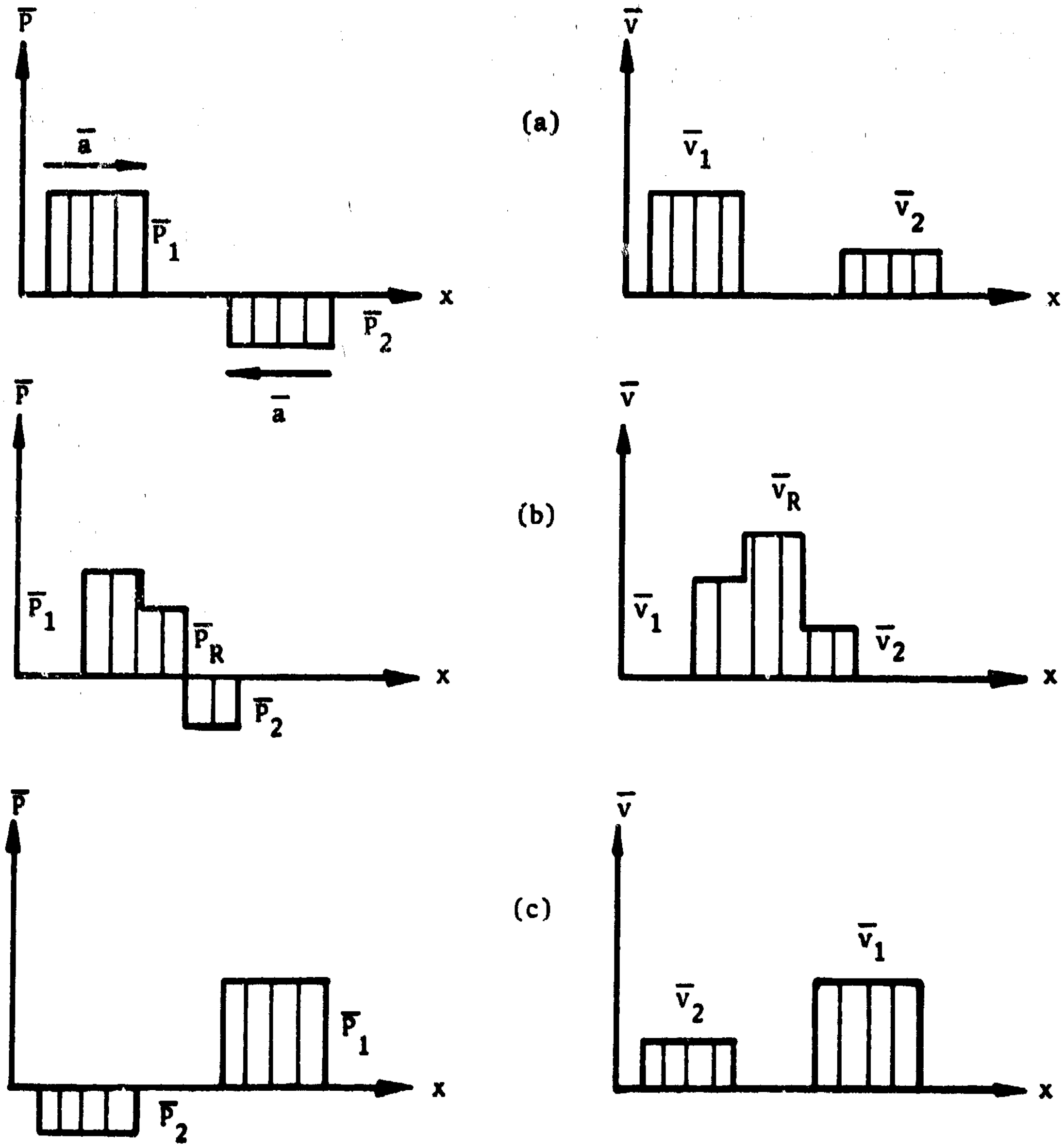


Figure B-3

One Compression and One Tension Wave Meet

a. Before meeting
b. At meeting
c. After meeting

3. Application of Wave Propagation Theory to Helical Springs

a. General Relationships

The expressions for particle velocity and wave propagation velocity derived for thin prismatical bars, as given by Eq's.(B-8) and (B-12), can be applied to helical springs, if one adapts the parameters m and k to the situation at hand. Thus one must define:

m = mass per unit length of spring wire along x-axis as defined by Figure A-1 of Appendix A.

k = stiffness constant, or spring constant per unit length of spring wire (also along x-axis), (see Eq.(A-1c)).

m is obtained with the help of the cross-sectional area A of the spring wire and the unit length along the x-axis. Thus:

$$m = \frac{A(1)\gamma}{g} = \frac{\pi d^2 \gamma}{4g} \tag{B-18}$$

The parameter k is derived with the help of the spring constant λ for the whole spring as given by Eq.(A-1a) in Appendix A:

$$\lambda = \frac{Gd^4}{8D^3 N} \tag{B-19}$$

Since the above is the spring constant for the full length L along the x-axis, any shortening of the spring increases the stiffness, one obtains k from:

$$k = \lambda L = \frac{Gd^4 L}{8D^3 N} = \frac{Gd^4 \pi}{8D^2} \tag{B-20}$$

b. Velocity of Wave Propagation or Surge Velocity in Helical Springs

When Eq's.(B-18) and (B-20) are substituted into Eq.(B-12) one obtains the following expression for the surge velocity:

$$a_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{\lambda L g}{\gamma\left(\frac{\pi d^2}{4}\right)}} \tag{B-21}$$

As in Eq.(A-18) this represents the surge velocity when the rotational inertia of the wire is neglected. This expression furnishes slightly large values, and it is best to use the corrected form of Eq.(A-19), i.e.

$$a = 0.99\sqrt{\frac{\lambda Lg}{\gamma(\frac{\pi d^2}{4})}} \qquad \text{(B-22)}$$

c. Particle Velocity in Helical Springs Without Driven Mass

Substitution of Eq's.(B-18) and (B-20) into Eq.(B-8), together with the expression relating the load P_f of the spring to its associated maximum shear stress τ_f, i.e.

$$P_f = \frac{\tau_f \pi d^3}{8D}$$

leads to an expression for the particle velocity as a function of the maximum shear stress:

$$v = \tau_f\sqrt{\frac{g}{2\gamma G}} \qquad \text{(B-23)}$$

The same expression results when one substitutes Eq.(B-21) into Eq.(A-43) to obtain $v_{max\ L}$. If one includes the rotational inertia term, the particle velocity becomes identical with the maximum velocity at x = L as given by Eq.(A-48):

$$v_{II} = 0.99\tau_f\sqrt{\frac{g}{2\gamma G}} \qquad \text{(B-24)}$$

Equations (B-22) and (B-24) are used throughout this report as the applicable design equations.

d. Maximum Attainable Velocity of Helical Spring Without Driven Mass.

Time of Separation from Support

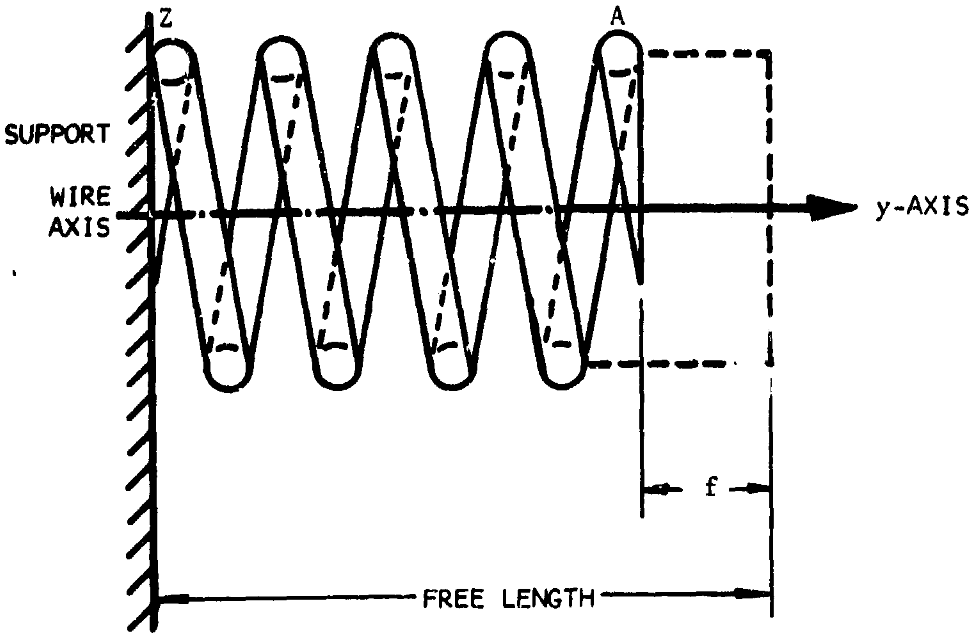


Figure B-4

Helical Spring

Figure B-4 shows a helical spring which is compressed a distance f along its y-axis by a corresponding axial force P_f. The spring is not rigidly fastened to the support at point Z. Contact is maintained only as long as there is a contact force. As soon as the contact force vanishes the spring will fly off the support. Note that L is the length of the wire along the x-axis.

When the force P_f is suddenly removed, a tensile wave containing a force $(-)P_f$ will start moving into the spring in the direction from point A to point Z. Associated with this relieving force is the particle velocity

$$v_H = 0.99\tau_f\sqrt{\frac{g}{2\gamma G}} = \frac{P_f}{\sqrt{mk}} \tag{B-25}$$

according to Eq.(B-24). As this tensile wave progresses all portions of the spring will acquire this velocity, and the axial force will become zero.

Once the particles at point Z have attained the above velocity, the spring will fly off its support. This will occur at the time

$$T_s = \frac{L}{a} \tag{B-26}$$

where a is the surge velocity according to Eq.(B-22) and T_s is called the surge time. Of course one starts counting time when the force P_f is removed from point A of the spring.

Since all particles have the same velocity when the spring separates from its support, i.e. there will be no strain energy locked into the spring, there cannot be any vibratory motion in the spring after separation.

e. Maximum Velocity of Spring Driven Mass

I. Period Between $t = 0$ and $t = 2T_s$

A. Derivation of Tensile Force and Particle Velocity Associated With Tension Wave

The following considers the events experienced by the spring mass system shown in Figure B-5 between the time $t = 0$, when the deflecting force P_f is removed from the mass M and thus a leftward moving tensile wave is started at point A, and the time $t = 2T_s = 2L/a$, when the front of this wave returns to point A after having been reflected at point Z at the other end of the spring.

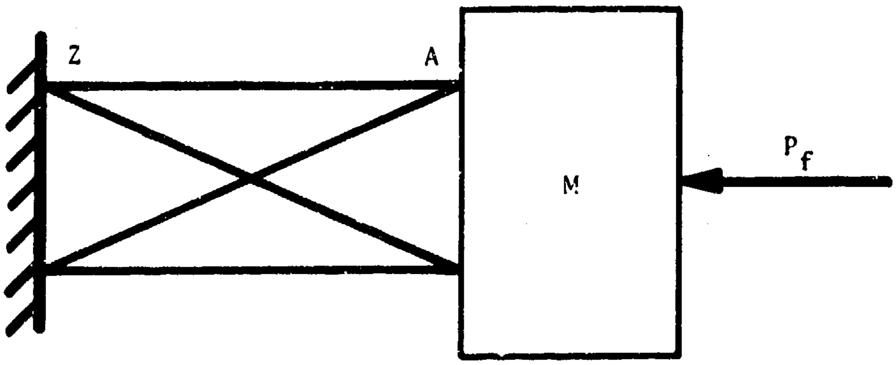


Figure B-5

Spring Driven Mass M

Since the effect of the removal of the deflecting force is communicated along the length L of the spring wire with the surge velocity a, one is justified in treating this situation in terms of the superposition of a constant compression of magnitude P_f and a leftward moving relieving tensile wave. The magnitude of the tensile force P(t) associated with this tensile wave will now be derived by considering the law of motion of mass M under the given circumstances. Figure B-6 shows a free body diagram of mass M at the time $t = 0$ when the deflecting force P_f has just been removed.

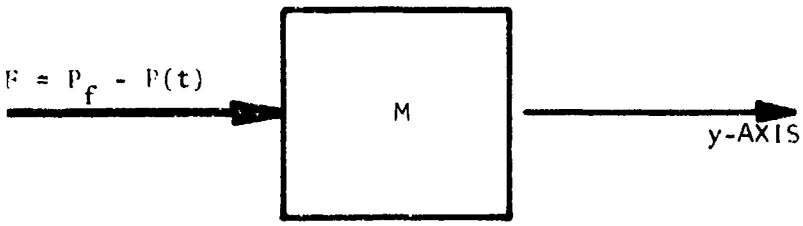


Figure B-6

Free Body Diagram of Mass After Deflecting Force has Been Removed

The only force acting on the mass is the contact force F, which consists of the vector sum of a force equal and opposite to the deflecting force P_f, (i.e. acting along the positive y-axis) and the relieving tensile force P(t) of the tension wave, which may be thought of as acting on the mass in the negative direction of the y-axis. Thus:

$$F = P_f - P(t) \tag{B-27}$$

The differential equation of motion according to Newton's Law becomes:

$$P_f - P(t) = M \frac{dv}{dt} \tag{B-28}$$

Now one considers that the velocity of the mass must be identical with the velocity of the particle at the contact point A as long as contact exists between the mass and the spring. The particle velocity is only a function of the tensile force P(t), and therefore it is expressed according to Eq.(B-8):

$$\frac{dv}{dt} = \frac{dP(t)}{dt}\sqrt{\frac{1}{mk}} \tag{B-29}$$

Substitution of Eq.(B-29) into Eq.(B-28) leads to:

$$P_f - P = \frac{M}{\sqrt{mk}} \frac{dP}{dt}$$

or

$$\frac{dP}{dt} + \frac{\sqrt{mk}}{M} P = P_f \frac{\sqrt{mk}}{M} \tag{B-30}$$

The complementary solution to Eq.(B-30) is obtained by means of the trial function

$$P_c = e^{st}$$

Substitution of this trial function and its derivative into the homogeneous part of Eq.(B-30) leads to the solution for the characteristic value s:

$$s = -\frac{\sqrt{mk}}{M} \tag{B-31}$$

The complementary solution is given by:

$$P_c = C_1 e^{-\frac{\sqrt{mk}}{M} t} \tag{B-32}$$

where C_1 is a constant which must be determined from the initial conditions. The particular solution of Eq.(B-30) is now obtained by the Method of Undetermined Coefficients. With the assumption that

$$P_p = C_2, \text{ and therefore } \frac{dP_p}{dt} = 0 \tag{B-33}$$

one obtains after substitution into Eq.(B-30):

$$P_p = P_f \tag{B-34}$$

The general solution is then given by the addition of Eq's.(B-32) and (B-33):

$$P(t) = C_1 e^{-\frac{\sqrt{mk}}{M} t} + P_f \tag{B-35}$$

It remains now to solve for the constant C_1 with the initial condition for P(t):

$$P(0) = 0 \tag{B-36}$$

i.e. at the instant of the release of the deflecting force P_f the relieving tension is still zero. Substitution of Eq.(B-36) into Eq.(B-35) leads to:

$$C_1 = - P_f \tag{B-37}$$

and the expression for the relieving tension force at point A is finally given by:

$$P(t) = P_f(1 - e^{-\frac{\sqrt{mk}}{M} t}) \tag{B-38}$$

The velocity of mass M, as well as the particle velocity at point A, is then for the period under consideration according to Eq.(B-8):

$$v_1 = \frac{P_f}{\sqrt{mk}}(1 - e^{-\frac{\sqrt{mk}}{M} t}) \tag{B-39}$$

This velocity is along the positive y-axis. The contact force, according to Eq.(B-27), becomes:

$$F = P_f - P(t) = P_f \, e^{-\frac{\sqrt{mk}}{M} t} \tag{B-40}$$

B. Desirable Change in Notation

To conform with the notation usually found in the literature dealing with wave propagation, one considers that according to Eq. (B-12):

$$a^2 = \frac{k}{m} \tag{B-41}$$

and thus

$$k = a^2 m \tag{B-42}$$

and accordingly

$$\sqrt{mk} = am \tag{B-43}$$

Now consider the exponent of Eq's.(B-38) to (B-40):

$$\frac{\sqrt{mk}}{M} = \frac{am}{M} = \frac{am(L)}{M(L)}$$

and since

$$mL = m_s \text{, the mass of the spring} \tag{B-44}$$

and further according to Eq.(B-26):

$$\frac{L}{a} = T_s$$

one may write:

$$\frac{\sqrt{mk}}{M} = \frac{m_s}{MT_s} \tag{B-45}$$

If one introduces

$$T = 2T_s \tag{B-46}$$

where T represents the time it takes for a surge wave to traverse the spring twice, then Eq.(B-45) becomes:

$$\frac{\sqrt{mk}}{M} = \frac{2m_s}{MT} \tag{B-47}$$

Now one introduces

$$\alpha = \frac{m_s}{M} \tag{B-48}$$

Thus finally one obtains

$$\frac{\sqrt{mk}}{M} = \frac{2\alpha}{T} \tag{B-49}$$

and Eq.(B-38) becomes:

$$P(t) = P_f(1 - e^{-\frac{2\alpha}{T}t}) \tag{B-50 a}$$

and Eq. (B-39) together with Eq. (B-25) becomes: $v_1 = v_H(1-e^{-\frac{2\alpha}{T}t})$ (B-50 b)

C. Separation of Spring from Support at Point Z

While the tensile wave travels from point A to point Z the total force $P_{t(1)}$ at any location will be given by

$$P_{t(1)} = - P_f + P(t) \tag{B-51}$$

A negative sign has been chosen for compression, i.e. P_f, while P(t) which represents tension, has been given a positive sign.

Whenever the wave arrives at an arbitrary place between points A and Z, the magnitude of P(t) corresponds to that at point A at t = 0, i.e. P = 0. The magnitude of P(t) at the same point at any time Δt later is found by computing the value of $P(\Delta t)$ at point A.

When the tensile wave arrives at point Z, it is reflected with the same sign since the particle velocity at this point must vanish as long as contact is maintained. (See also Section 2a of this appendix.)

The total force at point Z before separation has occurred, and after the wave has been reflected is given by:

$$P_{tZ} = - P_f + 2P(t) \tag{B-52}$$

If one now starts counting time at the instant the tensile wave arrives at point Z, Eq.(B-52) becomes with the help of equ. (B-50):

$$P_{tZ} = - P_f + 2P_f(1 - e^{-\frac{2\alpha}{T}t}) \tag{B-53}$$

The spring will leave the support at the time t_Z, after the tensile wave has arrived at point Z, when the force P_{tZ} of the last equation becomes zero. Thus one obtains for Eq.(B-53):

$$0 = P_f[-1 + 2(1 - e^{-\frac{2\alpha}{T}t_z})] \tag{B-54}$$

or

$$0 = 1 - 2e^{-\frac{2\alpha}{T} t_z}$$

This leads to

$$e^{\frac{2\alpha}{T} t_z} = 2$$

With the help of Eq's.(B-26) and (B-48) one obtains:

$$t_z = \frac{T}{2\alpha} \ln 2 = \frac{M}{m_s} T_s \ln 2 \tag{B-55}$$

To find the time of separation of the spring at point Z when time is counted from the moment the deflecting force is removed from the mass, one considers:

$$T_z = T_s + t_z = T_s\left(1 + \frac{M}{m_s} \ln 2\right) \tag{B-56}$$

Equation (B-56) is valid only for the mass ratios $M/m_s \leq 2.89$ and for $t < 3T_s$.

The time restriction is based on the fact that at $t = 3T_s$, counted from the removal of the deflecting force, a further reflection of the wave will take place at the point Z and Eq.(B-52) ceases to be a valid description.

This consideration is also responsible for the restriction on the mass ratio. Equation (B-55) is only valid for $t < 2T_s$, with the time counted from the instant of arrival of the wave at point Z. Therefore Eq.(B-55) must satisfy:

$$2T_s = \frac{2L}{a} \geq \frac{M}{m_s} \frac{L}{a} \ln 2 \tag{B-57}$$

Then

$$\frac{M}{m_s} \leq \frac{2}{\ln 2} = 2.89 \tag{B-58}$$

This means that whenever $M/m_s > 2.89$ the separation at point Z will take place at a time later than $t = 3T_s$, counted from the instant of the removal of the deflecting force P_f, and a new set of equations must be derived.

II. Period Between $t = 2T_s$ and $t = 4T_s$

Separation of Mass from Spring

A. Determination of Contact Force Between Mass and Spring

Separation of mass M from the spring at point A will occur when the contact force between them becomes zero. It is the purpose of the following to determine this time T_A, counted from the removal of the deflecting force. Since at this time the particle at point A will have attained its maximum velocity, it also represents the instant of maximum velocity of mass M.

Examine now the magnitude of the contact force at point A just an instant before the tensile wave, which has been reflected at point Z, arrives again at point A This occurs at $t < 2L/a = 2T_s = T$. According to Eq.(B-40), the magnitude of the contact force for $0 \leq t < T$:

$$F = P_f e^{-\frac{2\alpha}{T} t} \tag{B-59}$$

(With the above sign it represents the force of the spring on the mass in the direction of the positive y-axis.) Substitution of $t = T$, or rather a value of t just a little smaller than T, indicates that the contact force cannot vanish before the tensile wave is once again reflected at point A.

One need now to examine what happens at point A when $T \leq t < 2T$. The reflected tensile wave arrives at point A when $t = T$. Since there cannot be an abrupt change of the velocity of the mass, this incident tensile wave will be reflected as from a fixed end. This type of reflection causes a doubling of the tensile force of the incident wave at point A.

Note that there are then two waves moving from point A towards point Z:

a. the original tensile wave which was started by the removal of the deflecting force P_f, and
b. the wave which is reflected from A at $t = T$.

Furthermore there is the tensile wave moving from point Z toward point A.

In order to determine the contact force at A it is not only necessary to consider the action of all three of the above waves, but one must also take the initial compression of magnitude P_f into account.

Now let P_1 stand for the tensile force of the two waves moving away from point A. The wave which advances towards point A is merely the wave sent out from point A at $t = 0$, delayed a time T due to its travel across the spring and back. The tension produced due to this latter wave at point A is obtained by substituting $(t - T)$ for the time in Eq.(B-50), which represents the expression for the tension produced at point A in the preceding interval, (i.e. $0 \leq t < T$). Let this force be termed P_0:

(B-60)

The total tensile force at point A for the interval $T \leq t < 2T$ will thus be given by:

$$P(t) = P_1(t) + P_0(t - T) \tag{B-61}$$

The associated particle velocity is obtained as the difference between the particle velocity due to the tensile wave going away from point A and the one due to the wave moving towards point A. Thus according to Eq.(B-8):

$$v = \frac{1}{\sqrt{mk}}[P_1(t) - P_0(t - T)] \tag{B-62}$$

In order to determine the contact force F for the period under consideration, one proceeds in a manner similar to the one leading to Eq.(B-40). This means one must determine $P_1(t)$.

One starts by writing Newton's Equation of Motion for mass M again:

$$F = M\frac{dv}{dt} \tag{B-63}$$

(Refer to Figure B-6 for a free body diagram.) The contact force F consists of the original rightward acting force P_f as well as the sum of the leftward acting tensile forces given by Eq.(B-61). Equation (B-63) becomes:

$$F = P_f - P(t) = M\frac{dv}{dt} \tag{B-64}$$

Now substitute Eq.(B-61) and the derivative of the Eq.(B-62) into the above:

$$P_f - [P_1(t) + P_0(t - T)] = \frac{M}{\sqrt{mk}}\frac{d}{dt}[P_1(t) - P_0(t - T)] \tag{B-65}$$

With the help of Eq.(B-49) one obtains:

$$\frac{d}{dt}[P_1(t) - P_0(t - T)] + \frac{2\alpha}{T}[P_1(t) + P_0(t - T)] = \frac{2\alpha}{T}P_f \tag{B-66}$$

Equation (B-66) must now be solved for P(t) in order to find an expression for the contact force F.

Multiply all terms by $e^{\frac{2\alpha t}{T}}$

$$e^{\frac{2\alpha t}{T}}\frac{dP_1(t)}{dt} + \frac{2\alpha}{T}e^{\frac{2\alpha t}{T}}P_1(t) = e^{\frac{2\alpha t}{T}}\frac{dP_0(t - T)}{dt} + \frac{2\alpha}{T}P_0(t - T)e^{\frac{2\alpha t}{T}} - \frac{2\alpha}{T}P_0(t - T)e^{\frac{2\alpha t}{T}}$$

$$- \frac{2\alpha}{T}e^{\frac{2\alpha t}{T}}P_0(t - T) + \frac{2\alpha}{T}e^{\frac{2\alpha t}{T}}P_f \tag{B-67}$$

Note that $\pm\frac{2\alpha}{T}e^{\frac{2\alpha t}{T}} P_0(t - T)$ has been added to the right hand side of the above. Equation (B-67) may now be written as:

$$\frac{d}{dt}[e^{\frac{2\alpha t}{T}} P_1(t)] = \frac{d}{dt}[e^{\frac{2\alpha t}{T}} P_0(t - T)] - \frac{4\alpha}{T} e^{\frac{2\alpha t}{T}} P_0(t - T) + \frac{2\alpha}{T} e^{\frac{2\alpha t}{T}} P_f \tag{B-68}$$

Integration of the above leads to:

$$e^{\frac{2\alpha t}{T}} P_1(t) = e^{\frac{2\alpha t}{T}} P_0(t - T) - \frac{4\alpha}{T}\int e^{\frac{2\alpha t}{T}} P_0(t - T)\, dt + \frac{2\alpha}{T} P_f \int e^{\frac{2\alpha t}{T}} dt + C \tag{B-69a}$$

where C is a constant of integration. Now divide by $e^{\frac{2\alpha t}{T}}$

$$P_1(t) = P_0(t - T) - \frac{4\alpha}{T} e^{-\frac{2\alpha t}{T}}\int e^{\frac{2\alpha t}{T}} P_0(t - T)\, dt + \frac{2\alpha}{T} e^{-\frac{2\alpha t}{T}} P_f \int e^{\frac{2\alpha t}{T}} dt + Ce^{-\frac{2\alpha t}{T}} \tag{B-69b}$$

Now substitute Eq.(B-60) for $P_0(t - T)$:

$$P_1(t) = P_f - P_f e^{-2\alpha(\frac{t}{T} - 1)} - \frac{4\alpha}{T} e^{-\frac{2\alpha t}{T}}\int e^{\frac{2\alpha t}{T}} [P_f - P_f e^{-2\alpha(\frac{t}{T} - 1)}]\, dt + \frac{2\alpha}{T} e^{-\frac{2\alpha t}{T}} P_f \int e^{\frac{2\alpha t}{T}} dt + Ce^{-\frac{2\alpha t}{T}} \tag{B-70}$$

Rewrite the above:

$$P_1(t) = P_f - P_f e^{-2\alpha(\frac{t}{T} - 1)} - \frac{4\alpha}{T} e^{-\frac{2\alpha t}{T}}\int P_f e^{\frac{2\alpha t}{T}} dt + \frac{4\alpha}{T} e^{-\frac{2\alpha t}{T}}\int P_f e^{2\alpha} dt + e^{-\frac{2\alpha t}{T}}\int \frac{2\alpha}{T} P_f e^{\frac{2\alpha t}{T}} dt + Ce^{-\frac{2\alpha t}{T}} \tag{B-71}$$

After the indicated integrations are carried out, one obtains:

$$P_1(t) = P_f e^{-2\alpha(\frac{t}{T} - 1)} [\frac{4\alpha t}{T} - 1] + Ce^{-\frac{2\alpha t}{T}} \tag{B-72}$$

To evaluate the constant C in the above one needs $P_1(T)$. To obtain it the following reasoning is used: The wave which was started at $t = 0$ has just reached point A, and it has been reflected. Since the magnitude of P(0), according to Eq.(B-50), is zero, both the incident as well as the reflected wave have zero tension at $t = T$. The only force present is due to the P(T), i.e. again according to Eq.(B-50):

$$P_1(T) = P(T) = P_f(1 - e^{-\frac{2\alpha T}{T}}) \tag{B-73}$$

Substitution of the above into Eq.(B-72) gives:

$$P_f - P_f e^{-2\alpha} = P_f e^{-2\alpha(\frac{T}{T} - 1)} [\frac{4\alpha T}{T} - 1] + Ce^{-2\alpha} \tag{B-74}$$

at $t = T$. Solving for the constant one obtains:

$$C = P_f \frac{(2 - 4\alpha - e^{-2\alpha})}{e^{-2\alpha}} \tag{B-75}$$

Finally:

$$P_1(t) = P_f[e^{-2\alpha(\frac{t}{T} - 1)} (\frac{4\alpha}{T} t - 1) + (\frac{2 - 4\alpha - e^{-2\alpha}}{e^{-2\alpha}})e^{-\frac{2\alpha}{T} t}] \tag{B-76}$$

The contact force F may now be determined with the help of the left hand side of Eq.(B-65), i.e.:

$$F = P_f - P_1(t) - P_0(t - T) \tag{B-77}$$

Substitution of Eq's.(B-76) and (B-60) leads to:

$$F = P_f e^{-2\alpha(\frac{t}{T} - 1)} [e^{-2\alpha} - 4\alpha(\frac{t}{T} - 1)] \tag{B-78}$$

B. Determination of the Separation Time T_A

The separation time T_A is determined by means of setting

$$F(T_A) = 0 \tag{B-79}$$

into Eq.(B-78). One obtains:

$$0 = e^{-2\alpha} - 4\alpha(\frac{T_A}{T} - 1) \tag{B-80}$$

since the factor $P_f e^{-2\alpha(\frac{T_A}{T} - 1)}$ cannot be zero at $t = T_A$. Finally from Eq.(B-80):

$$T_A = T(1 + \frac{e^{-2\alpha}}{4\alpha}) \tag{B-81}$$

According to Eq's.(B-46) and (B-48) respectively:

$$T = 2T_s \quad \text{and} \quad 2\alpha = \frac{2m_s}{M}$$

so that Eq.(B-81) becomes:

$$T_A = T_s(2 + \frac{M}{2m_s} e^{-\frac{2m_s}{M}}) \tag{B-82}$$

C. Determination of Maximum Velocity of Mass

Substitution of Eq's.(B-60) and (B-76) into Eq.(B-62) furnishes an expression for the velocity of the mass M while contact is maintained with the spring at point A. Thus

$$v = \frac{P_f}{\sqrt{mk}}[e^{-2\alpha(\frac{t}{T} - 1)}(\frac{4\alpha}{T} t + 2 - 4\alpha - e^{-2\alpha}) - 1] \tag{B-83}$$

Now consider that according to Eq.(B-8), and its modified form Eq.(B-25) when applied to a helical spring:

$$\frac{P_f}{\sqrt{mk}} = 0.99\tau_f\sqrt{\frac{g}{2\gamma G}} = v_H$$

Thus Eq.(B-83) becomes:

$$v_2 = v_H[e^{-2\alpha(\frac{t}{T} - 1)}(\frac{4\alpha}{T} t + 2 - 4\alpha - e^{-2\alpha}) - 1] \tag{B-84}$$

To obtain the maximum velocity of the mass one evaluates the above at $t = T_A$, the time of separation as determined by Eq.(B-81):

$$v_{max} = v_H[-1 + 2e^{-\frac{e}{2}^{-\frac{2m_s}{M}}}] \tag{B-85}$$

Figure B-7 shows a tabulation of the term in brackets in Eq. (B-85).

Since the expression for the contact force F of Eq. (B-64), which forms the basis of the present section, is valid only for $2T_S \leq t < 4T_S$ there will also be a limitation on the ratio M/m_s. T_A must be less than $4T_S$, and therefore substitution into Eq. (B-82) leads to:

$$4T_s > T_s\left(2+\frac{M}{2m_s}e^{-\frac{2m_s}{M}}\right)$$

This in turn requires that:

$$4\,\frac{m_s}{M} > e^{-\frac{2m_s}{M}} \qquad \text{(B-86)}$$

This condition can only be fulfilled for $M/m_s \leq 5.69$.

$\frac{M}{m_s}$	Value of Factor	$\frac{M}{m_s}$	Value of Factor
0.0	1.00000	2.8	0.56577
0.1	1.00000	2.9	0.55624
0.2	0.99995	3.0	0.54719
0.3	0.99873	3.1	0.53858
0.4	0.99327	3.2	0.53038
0.5	0.98177	3.3	0.52257
0.6	0.96464	3.4	0.51512
0.7	0.94338	3.5	0.50801
0.8	0.91958	3.6	0.50121
0.9	0.89452	3.7	0.49471
1.0	0.86914	3.8	0.48848
1.1	0.84409	3.9	0.48252
1.2	0.81977	4.0	0.47681
1.3	0.79641	4.1	0.47132
1.4	0.77415	4.2	0.46605
1.5	0.75304	4.3	0.46099
1.6	0.73307	4.4	0.45613
1.7	0.71423	4.5	0.45144
1.8	0.69647	4.6	0.44693
1.9	0.67974	4.7	0.44258
2.0	0.66397	4.8	0.43839
2.1	0.64911	4.9	0.43435
2.2	0.63510	5.0	0.43045
2.3	0.62187	5.1	0.42668
2.4	0.60938	5.2	0.42303
2.5	0.59757	5.3	0.41951
2.6	0.58639	5.4	0.41610
2.7	0.57581		

Figure B-7

Tabulation of Factor $- 1 + 2e^{-\frac{M}{2}e^{\frac{-2m_s}{M}}}$

(See equ. B-85)

APPENDIX C

DISTANCE TRAVELLED BY MASS M BETWEEN TIME OF SPRING RELEASE AND TIME OF SEPARATION OF MASS FROM SPRING

The results of Appendix B are now used to determine the distance through which mass M must travel until it reaches its maximum velocity. This is accomplished by integrating the velocity of the mass with respect to time between $t = 0$, when the spring is released, and $t = T_A$, when the maximum velocity of the mass is attained.

Equation (B-39) of Appendix B shows that the velocity of the mass during the time interval $0 \leq t < 2T_S = T$ is given by:

$$v_1 = \frac{P_f}{\sqrt{mk}} (1 - e^{-\frac{\sqrt{mk}}{M}t}) \tag{C-1}$$

Using the change of notation indicated on page B-14, and letting

$$\frac{P_f}{\sqrt{mk}} = v_H \tag{C-2}$$

according to Eq's.(B-8) and (B-25), one may express Eq.(C-1) in the following form:

$$v_1 = v_H(1 - e^{-\frac{2\alpha}{T}t}) \tag{C-3}$$

where

$\alpha = \frac{m_S}{M}$ the ratio of spring mass and driven mass

$T = 2T_S$, twice the surge time T_S .

The velocity of the mass for the time interval $T \leq t < 2T = 4T_S$ is given by Eq.(B-84) of Appendix B:

$$v_2 = v_H[e^{-2\alpha(\frac{t}{T} - 1)} (\frac{4\alpha}{T}t + 2 - 4\alpha - e^{-2\alpha}) - 1] \tag{C-4}$$

1. Distance Traveled by Mass M During Interval from t = 0 to t = T

Integration of Eq.(C-3) furnishes the distance F_1, which is traveled by the mass between $t = 0$ and $t = T$. Thus:

$$F_1 = v_H \int_0^T (1 - e^{-\frac{2\alpha}{T}t}) \, dt \tag{C-5}$$

Performing the integration between the indicated limits leads to:

$$F_1 = v_H T[1 + \frac{1}{2\alpha}(e^{-2\alpha} - 1)] \tag{C-6}$$

2. Distance Travelled by Mass M During Interval from $t = T$ to $t = T_A$

Integration of Eq.(C-4) between the limits of $t = T$ and $t = T_A$ furnishes the distance F_2 travelled during this time interval. After rearrangement one obtains:

$$F_2 = v_H e^{2\alpha}\int_T^{T_A} [\frac{4\alpha}{T} t e^{-\frac{2\alpha}{T}t} + (2 - 4\alpha - e^{-2\alpha}) e^{-\frac{2\alpha}{T}t} - \frac{1}{e^{2\alpha}}] \, dt \tag{C-7}$$

where, according to Eq.(B-82):

$$T_A = T(1 + \frac{1}{4\alpha}e^{-2\alpha})$$

Term by term integration furnishes the following results:

$$F_{2(a)} = v_H e^{2\alpha}\,\frac{4\alpha}{T}\int_T^{T_A} t e^{-\frac{2\alpha}{T}t} \, dt = \frac{v_H T}{\alpha}[2\alpha+1-e^{-\frac{e^{-2\alpha}}{2}}(4\alpha+e^{-2\alpha}+2)] \tag{C-8}$$

and

$$\begin{aligned} F_{2(b)} &= v_H e^{2\alpha}(2 - 4\alpha - e^{-2\alpha})\int_T^{T_A} e^{-\frac{2\alpha}{T}t} \, dt \\ &= \frac{v_H T}{2\alpha}(4\alpha + e^{-2\alpha} - 2)(e^{-\frac{e^{-2\alpha}}{2}} - 1) \end{aligned} \tag{C-9}$$

Furthermore:

$$F_{2(c)} = - v_H \int_T^{T_A} dt = - \frac{v_H T}{4\alpha}e^{-2\alpha} \tag{C-10}$$

Addition of Eq's.(C-8), (C-9) and (C-10) leads to:

$$F_2 = \frac{v_H T}{\alpha}(2 - 2e^{-\frac{e^{-2\alpha}}{2}} - \frac{3}{4}e^{-2\alpha}) \tag{C-11}$$

3. Total Distance Travelled by Mass M During Interval from $t = 0$ to $t = T_A$

The total distance F_t travelled by mass M during the interval from $t = 0$ to $t = T_A$ is now obtained by addition of Eq's.(C-6) and (C-11):

$$F_t = F_1 + F_2 = v_H T\left[1 + \frac{1}{2\alpha}\left(3 - \frac{1}{2}e^{-2\alpha} - 4e^{-\frac{e^{-2\alpha}}{2}}\right)\right] \qquad \text{(C-12)}$$

It is convenient to express the product $v_H T$ in terms of the spring deflection f. Equation (A-43) gives:

$$v_H = \frac{fa}{L} = \frac{f}{T_S} = \frac{2f}{T}$$

Therefore Eq.(C-12) may be written in the following form:

$$F_t = fC_f \qquad \text{(C-13)}$$

where

$$C_f = 2 + \frac{1}{\alpha}\left(3 - \frac{e^{-2\alpha}}{2} - 4e^{-\frac{e^{-2\alpha}}{2}}\right) \qquad \text{(C-14)}$$

APPENDIX D

EXPERIMENTAL INVESTIGATION

1. Aims of Experiment

The present appendix describes the experimental phase of the investigation. This experimentation had the following aims:

a. Verification of the "Erwood Curves" (see Figure D-1) which give the reliability of the M42G primer in terms of percentage of primers fired versus firing pin velocity (at ambient temperature). These curves are plotted for firing pin velocities between 50 and 300 in/sec with firing pin kinetic energy levels of 16, 14, 12, 10, 9, 6 and 5 inch-ounces respectively.

b. Extension of the above mentioned M42G primer firing curves to the highest possible velocities attainable by firing pins of comparable energy levels which are driven by helical compression springs. This was to be obtained at both ambient temperature as well as -40 degrees F. The energy levels were to be 16, 14, 12, 10, 8, 6 and 5 inch-ounces. In addition the "Erwood" range was also to be tested at -40 degrees F.

c. Since the test setup was to use helical compression springs, the experiment could also serve as a verification of the theoretical results concerning the velocities of the spring driven masses. (See Appendix B.)

The following sections discuss the test program, the test setup, as well as the test results.

2. Test Program

Figure D-2 represents the test program. It gives the various levels of kinetic energy of the firing pins as well as their masses and velocities at which the primers were to be tested at both ambient temperature as well as -40 degrees F. (The firing pin identification numbers are indicated in the upper left hand corner of each box.) The velocities range from 50 to 1200 in/sec. The top velocity of 1200 in/sec was chosen because computation, using the theory of Appendix B, had indicated that this speed was close to the ultimate one attainable for such a system. (See computations in Part b below.)

The determination of the firing pin masses will now be shown on hand of an example: Let it be required to find the mass of a firing pin for an energy level of 10 inch-ounces and a velocity of 200 in/sec. Consider that

$$\frac{K.E.}{16} = \frac{1}{2}mv^2 \qquad \text{(D-1)}$$

where

K.E. = kinetic energy in inch-ounces
m = firing pin mass ($lb\text{-}sec^2/in$)
v = firing pin velocity (in/sec)

Then according to the above:

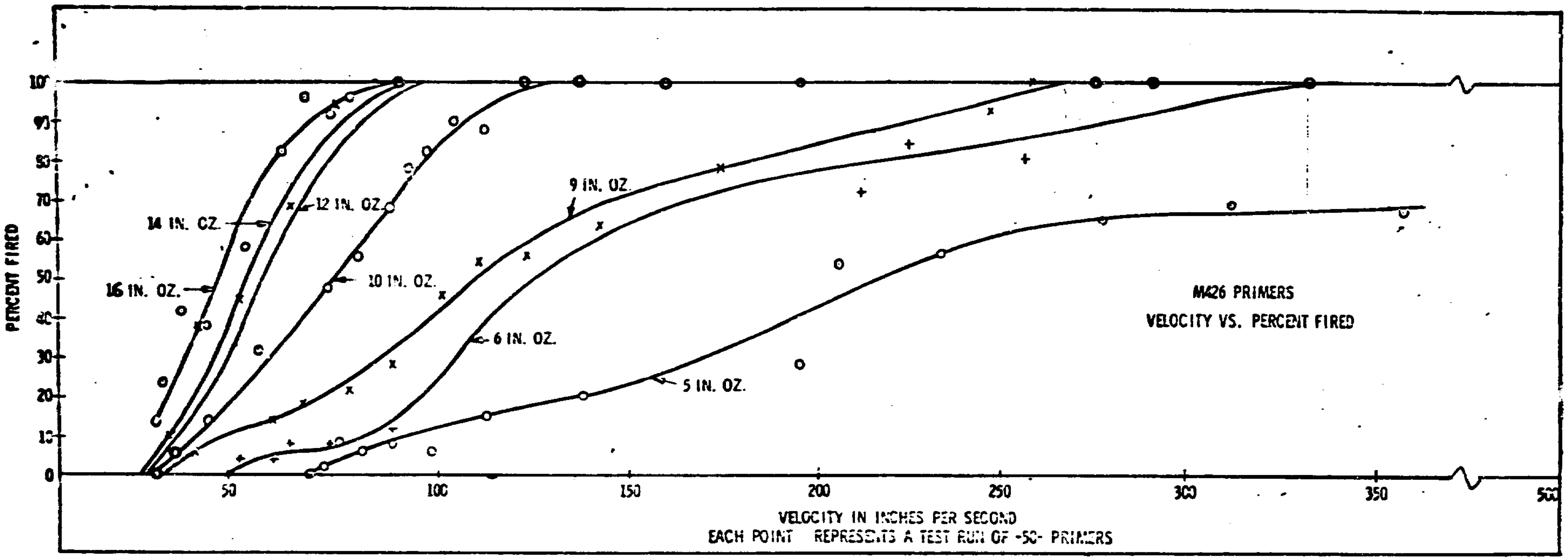


Figure D-1

M42G Primer Firing Curves

(From: Report of May 10,1956
by Erwood Corp., Chicago
N.B.S. Contract DAL-49-186-502-ORD(p)-199)

Velocity of Firing Pin (in/sec)	Mass of Firing Pin at 16 in.oz.	Ident. No. 14 in.oz.	12 in.oz.	10 in.oz.	8 in.oz.	6 in.oz.	5 in.oz.
50			1 600.0	2 500.0	3 400.0		
100	4 200.0	5 175.0	6 150.0	7 125.0	8 100.0	9 75.0	10 62.5
150	11 88.8889	12 77.7778	13 66.6667	14 55.5556	15 44.4445	16 33.3334	17 27.7778
200	18 50.0	19 43.75	20 37.5	21 31.25	22 25.0	23 18.75	24 15.625
250	25 32.0	26 28.0	27 24.0	28 20.0	29 16.0	30 12.0	31 10.0
300	32 22.2223	33 19.4444	34 16.6667	35 13.8889	36 11.1112	37 8.3334	38 6.9444
350	39 16.3265	40 14.285	41 12.2448	42 10.204	43 8.1632	44 6.1224	45 5.1020
400	46 12.5	47 10.9375	48 9.375	49 7.8125	50 6.25	51 4.6875	52 3.9062
500	53 8.0	54 7.0	55 6.0	56 5.0	57 4.0	58 3.0	59 2.5
600	60 5.5555	61 4.8611	62 4.1666	63 3.4722	64 2.7777	65 2.0833	66 1.7361
700	67 4.0816	68 3.5714	69 3.0612	70 2.5510	71 2.0408	72 1.5306	73 1.2755
800	74 3.125	75 2.7343	76 2.3437	77 1.9531	78 1.5625	79 1.1718	80 0.9765
900	81 2.4691	82 2.1604	83 1.8518	84 1.5432	85 1.2345	86 0.9259	87 0.7716
1000	88 2.0	89 1.75	90 1.50	91 1.25	92 1.0	93 0.75	94 0.625
1100	95 1.6528	96 1.4462	97 1.2396	98 1.0330	99 0.8264	100 0.6198	101 0.5165
1200	102 1.3888	103 1.2152	104 1.0416	105 0.8680	106 0.6944	107 0.5208	108 0.4340

Figure D-2

Test Program for M42G Primer Firing Tests
(Masses of Firing Pins in $lb\text{-}sec^2/in \times 10^6$)

$$m = \frac{K.E.}{8v^2} = \frac{10}{8\ (200)^2} = 31.25 \times 10^{-6}\ (lb\text{-}sec^2/in) \qquad (D-2)$$

3. Test Setup

Figure D-3 shows an exploded view of the test apparatus. The M42G primers are mounted in a .38 caliber cartridge, and this cartridge is inserted into the cartridge holder. This holder may be fastened into the front of the tube. Before its release, the firing pin is held against the pull pin by the compression of the spring. This compression is made adjustable by the leadscrew slider. A scale (not shown) which is mounted on one of the runway cover plates makes it possible together with the micrometric dial to repeat the spring setting accurately.

When the firing pin is released, the spring is allowed to expand freely until it flies off the supporting lever. To prevent the spring from following the separated firing pin it is stopped by the end of the slot in which the pad rides. (The spring leaves its support some time after the firing pin separates from the spring. See Appendix B.) It is still to be noted that the spring pad is fixed to the spring.

Since it was necessary to divide the firing pins into five groups of different diameters, all parts which depend on the firing pin diameter such as tubes, levers, etc. had to be made in sets of five. In addition, in order to make the testing as fast as possible thirty cartridge holders were made.

The velocity of the firing pin is measured with the help of a Fotonic Sensor Model KD-45 Serial 65 (Mechanical Technology Inc., Latham, N.Y.). The sensing probe(s) of this unit consist of 0.082 inch bundles of approximately 800 optical fibers. One half of these fibers transmit a light source, while the other half may again receive the light source once it has been suitably reflected. The output voltage of this unit is proportional to the quantity of the light reflected, and is used to trigger an electronic counter (Beckman Universal Eput and Timer, Model No. 7360).

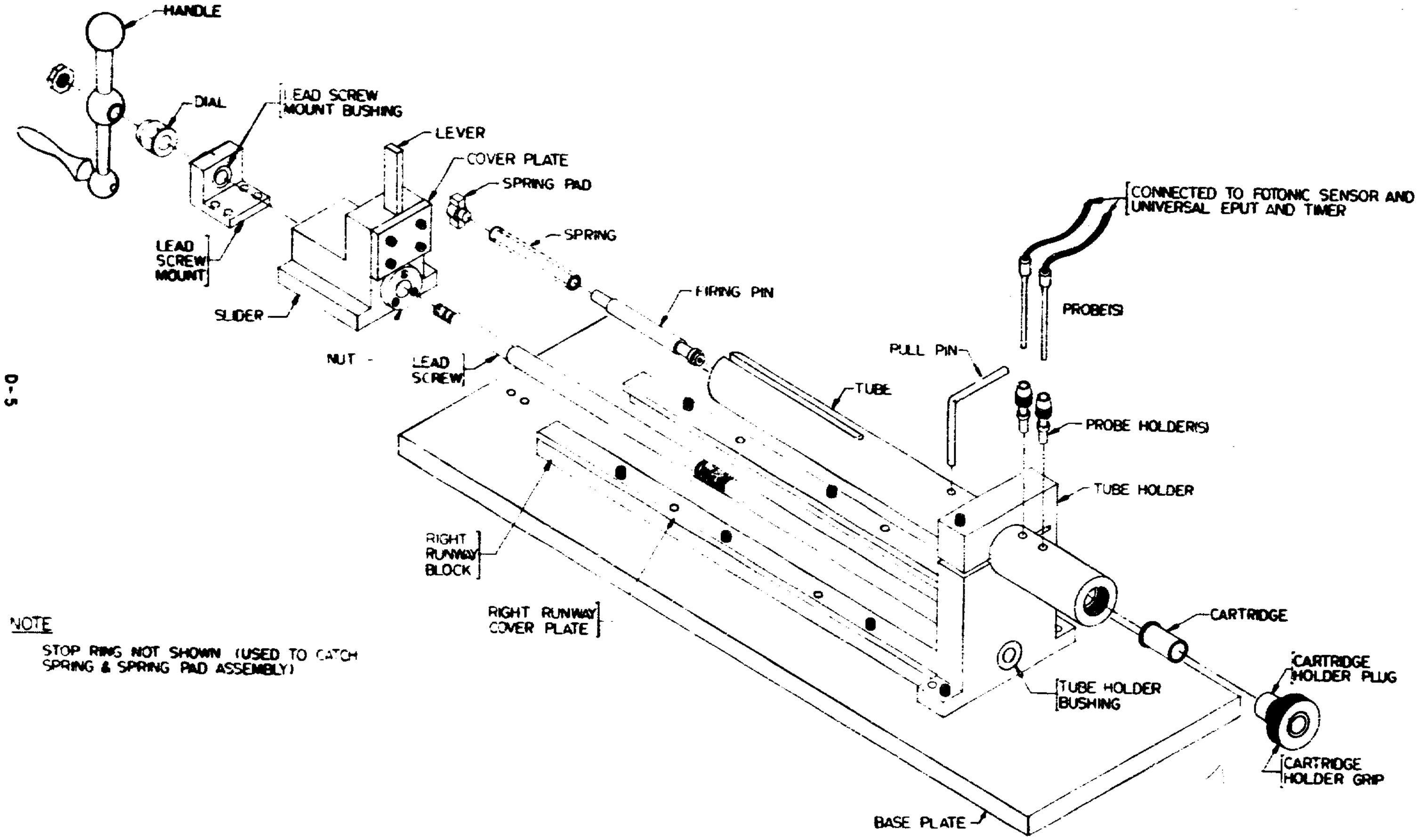
Depending on the size of the firing pin, a single probe or a two probe technique of measuring the velocity of the firing pin is used. Both methods of measuring are described in Section b below.

a. Design of Firing Pins

In order to obtain the firing pin masses described by the test program it was necessary to divide them into five groups of different diameters:

Group I:	0.500 inch diameter
Group II:	0.312 inch diameter
Group III:	0.187 inch diameter
Group IV:	0.125 inch diameter
Group V:	0.093 inch diameter

Further subdivision according to shape became necessary in order to accomodate the velocity measurements. Figures D-4, D-5a, D-5b, D-5c, D-5d, D-6a, D-6b, D-6c, D-7a, D-7b, D-7c, D-8a, D-8b and D-8c show all the firing pins used in the experiment. The masses are given in $lb \cdot sec^2/in$ and in grams and all relevant dimensions are given in inches. In all cases the pins were made of



NOTE

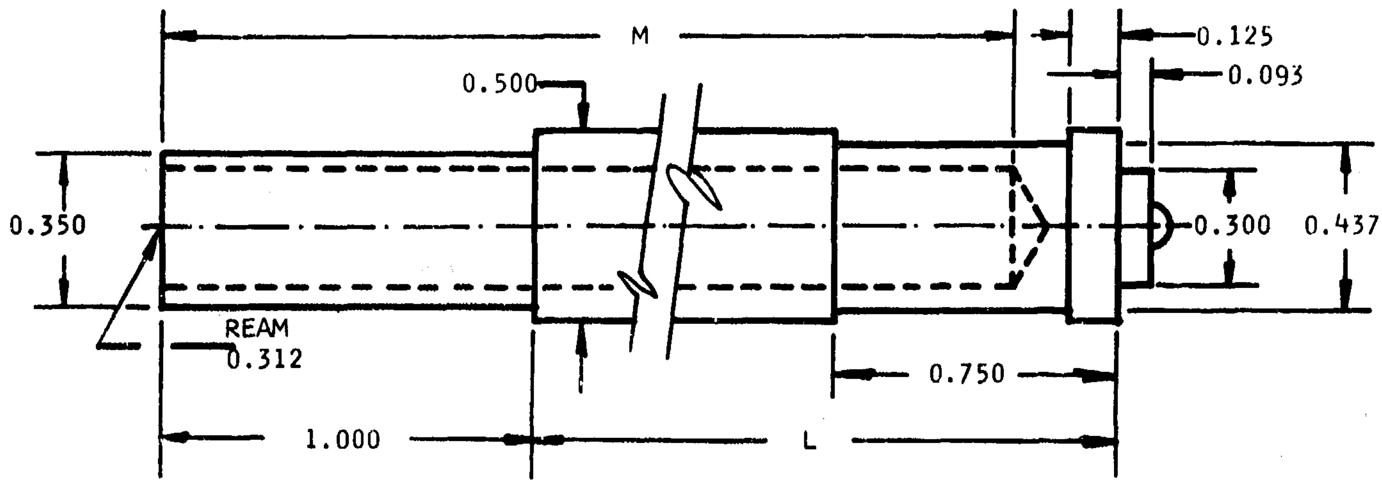
STOP RING NOT SHOWN (USED TO CATCH SPRING & SPRING PAD ASSEMBLY)

FIGURE D-3

PRIMER FIRING TEST SETUP

3-21-69	PRIMER FIRING TEST, ASSEMBLY	
HALF	THE CITY COLLEGE SCHOOL OF TECHNOLOGY NEW YORK, N.Y.	A-1
WGJ		

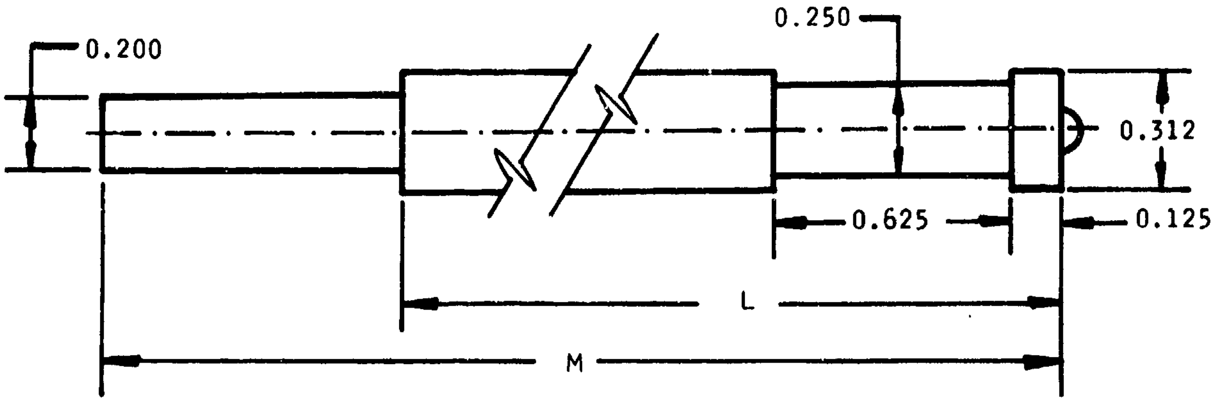
steel. Their tips consisted of the usual hemispheres of .030 inch radius. All firing pin masses were held within ± 0.5 percent of their theoretical weights.



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
1	600 x 10^{-6}	105.077	4.042	-
2	500	87.564	3.346	-
3	400	70.051	2.648	-
4	200	35.025	2.305	2.930
5	175	30.647	2.020	2.645

Figure D-4

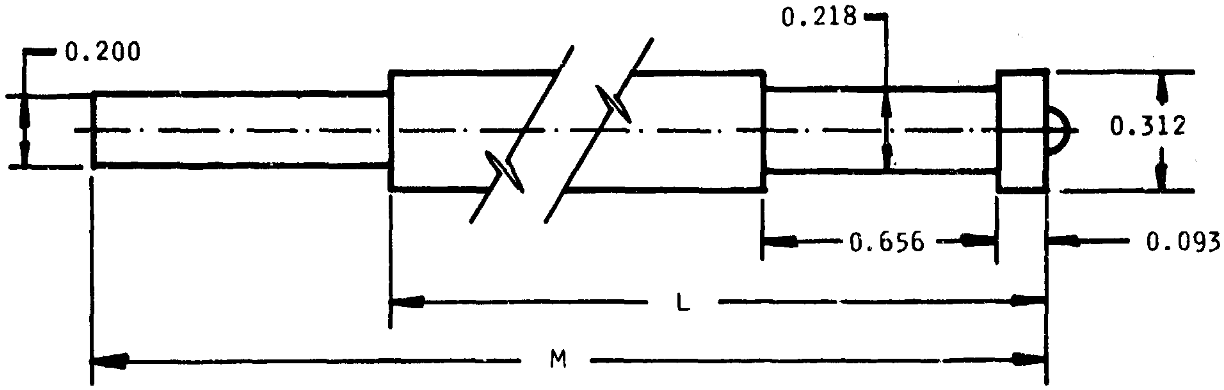
Group I Firing Pins



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
6	150 x 10^{-6}	26.269	2.817	3.127
7	125	21.891	2.372	2.875
8	100	17.513	1.927	2.427
9	75	13.134	1.483	2.000
10	62.5	10.945	1.263	1.763
11	88.8889	15.567	1.730	2.230
12	77.7778	13.621	1.532	2.032
13	66.6667	11.675	1.335	1.835
14	55.5556	9.729	1.137	1.637
15	44.4445	7.783	0.937	1.439
18	50.0000	8.756	1.037	1.537

Figure D-5a

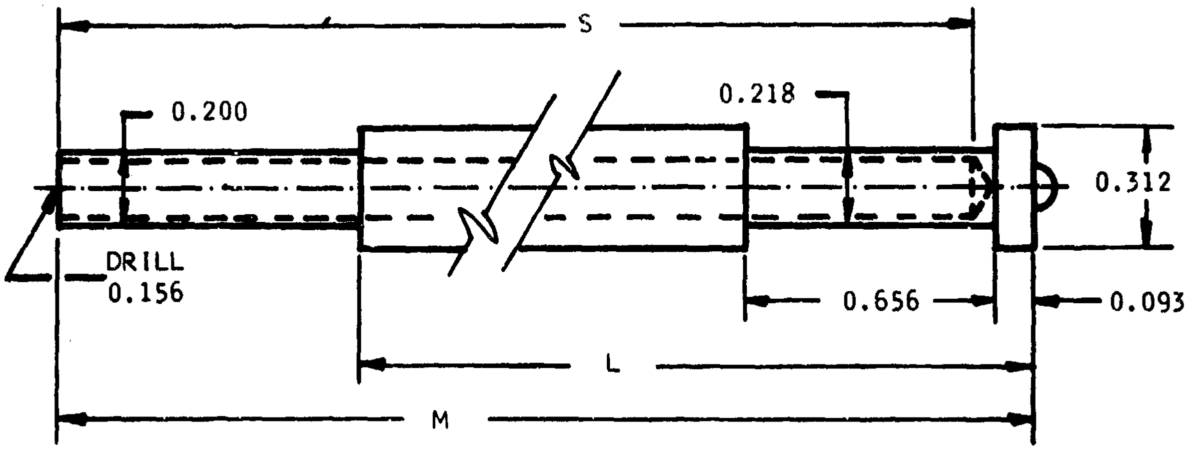
Group II-1 Firing Pins



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
16	33.3334×10^{-6}	5.837	0.902	1.417
19	43.75	7.661	1.087	1.602
20	37.50	6.567	0.976	1.491
21	31.25	5.472	0.865	1.380
25	32.00	5.604	0.878	1.393

Figure D-5b

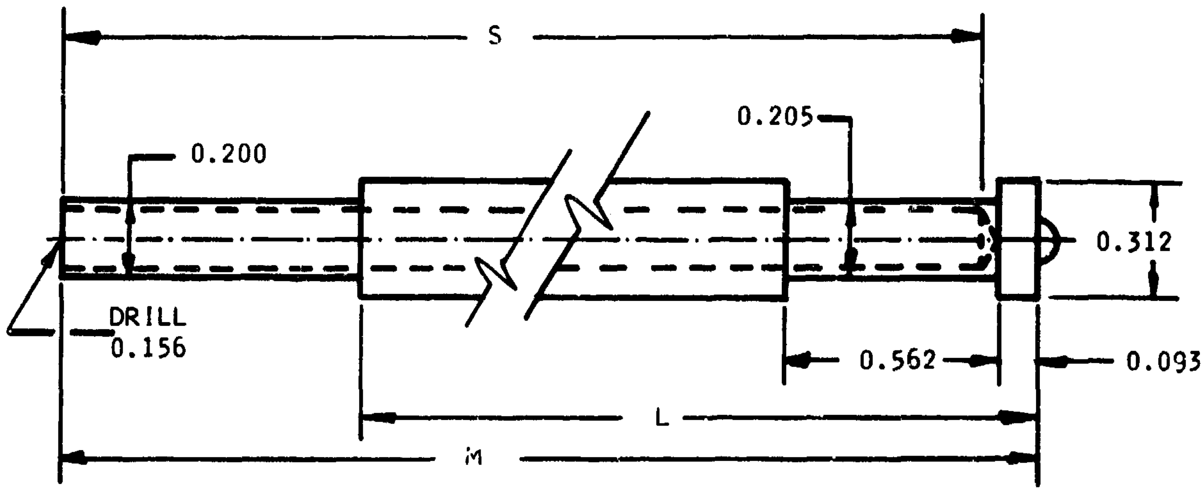
Group II-2 Firing Pins



I.D. No.	Mass		L	M	S
	lb-sec^2/in	gram			
17	27.778×10^{-6}	4.864	1.061	1.581	1.488
22	25.000	4.378	0.995	1.515	1.422
23	18.75	3.283	0.846	1.366	1.273
26	28.00	4.903	1.066	1.586	1.493
27	24.00	4.203	0.971	1.491	1.398
28	20.00	3.502	0.876	1.396	1.303
32	22.2223	3.891	0.928	1.448	1.355
33	19.4444	3.405	0.863	1.383	1.290

<u>Figure D-5c</u>

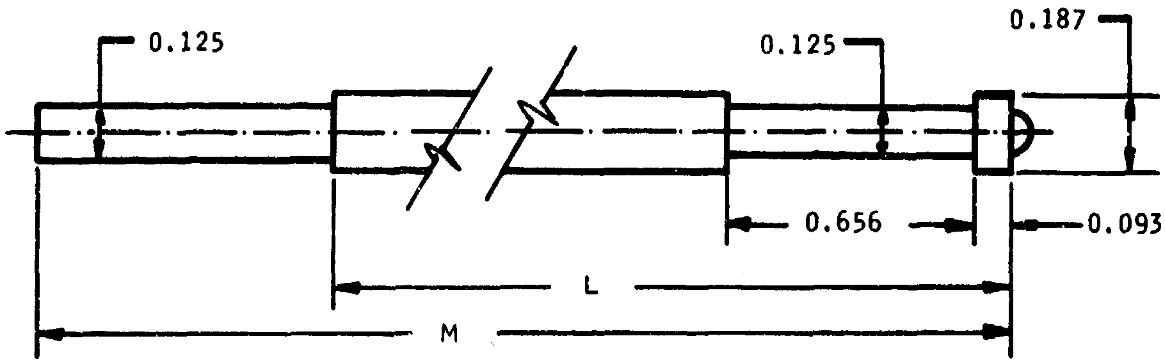
<u>Group II-3 Firing Pins</u>



I.D. No.	Mass		L	M	S
	lb-sec^2/in	gram			
24	15.625×10^{-6}	2.736	0.760	1.260	1.167
29	16.000	2.802	0.769	1.269	1.176
34	16.6667	2.918	0.785	1.285	1.192
39	16.3265	2.859	0.776	1.276	1.183

Figure D-5d

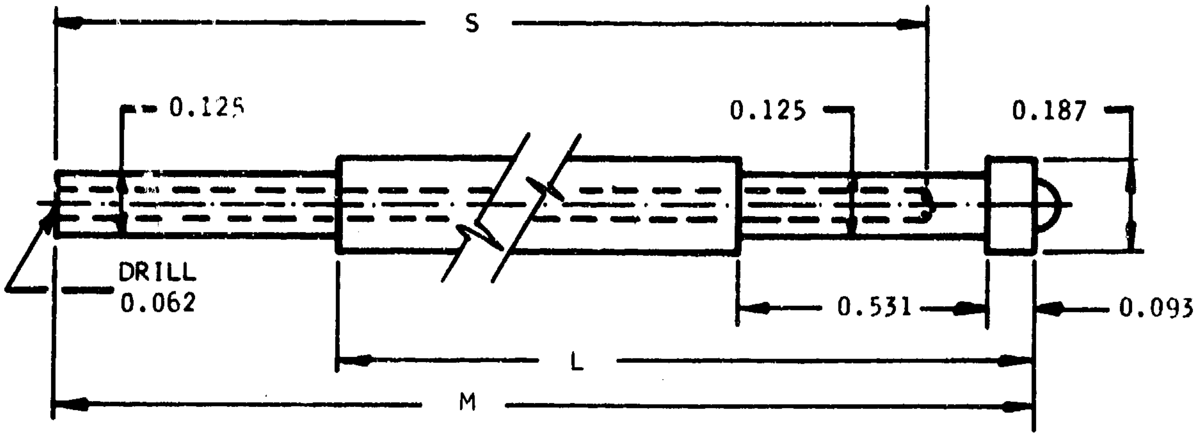
Group II-4 Firing Pins



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
30	12.000x10^{-6}	2.101	0.902	1.402
35	13.8889	2.432	0.995	1.495
36	11.1112	1.945	0.858	1.358
40	14.285	2.501	1.015	1.515
41	12.2448	2.144	0.914	1.414
46	12.500	2.189	0.926	1.426
47	10.9375	1.915	0.849	1.349

Figure D-6a

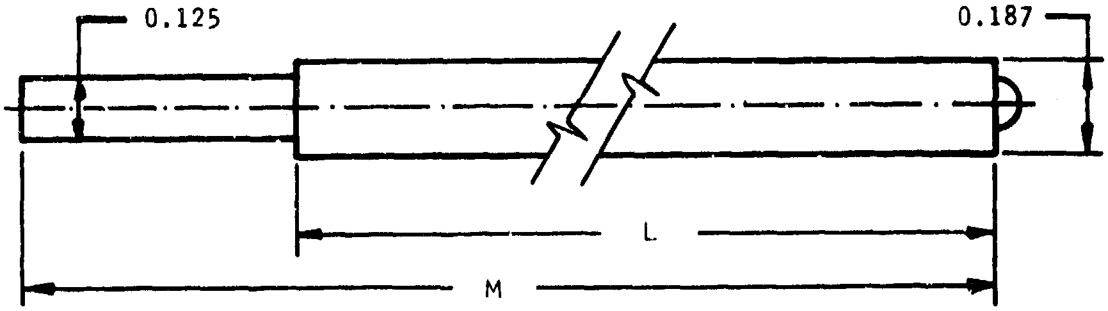
Group III-1 Firing Pins



I.D. No.	Mass		L	M	S
	lb-sec²/in	gram			
31	10.000×10^{-6}	1.751	0.853	1.353	1.260
37	8.3334	1.459	0.760	1.260	1.167
42	10.204	1.787	0.864	1.364	1.271
43	8.1632	1.429	0.751	1.251	1.158
48	9.375	1.641	0.818	1.318	1.225
49	7.8125	1.36[illegible]	0.731	1.231	1.138
53	8.0000	1.401	0.741	1.241	1.148

Figure D-6b

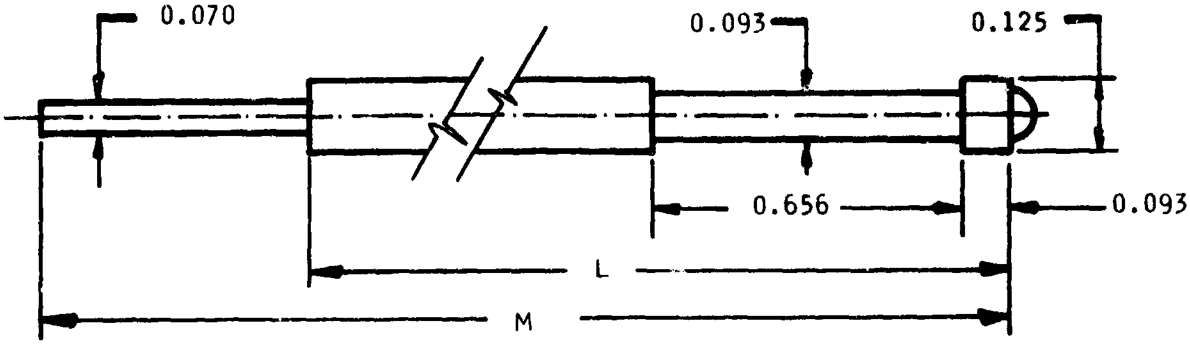
Group III-2 Firing Pins



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
38	6.9444×10^{-6}	1.216	0.315	0.690
44	6.1224	1.072	0.275	0.650
50	6.250	1.094	0.281	0.656
54	[illegible]	1.225	0.318	0.693
55	6.000	1.050	0.269	0.644

Figure D-6c

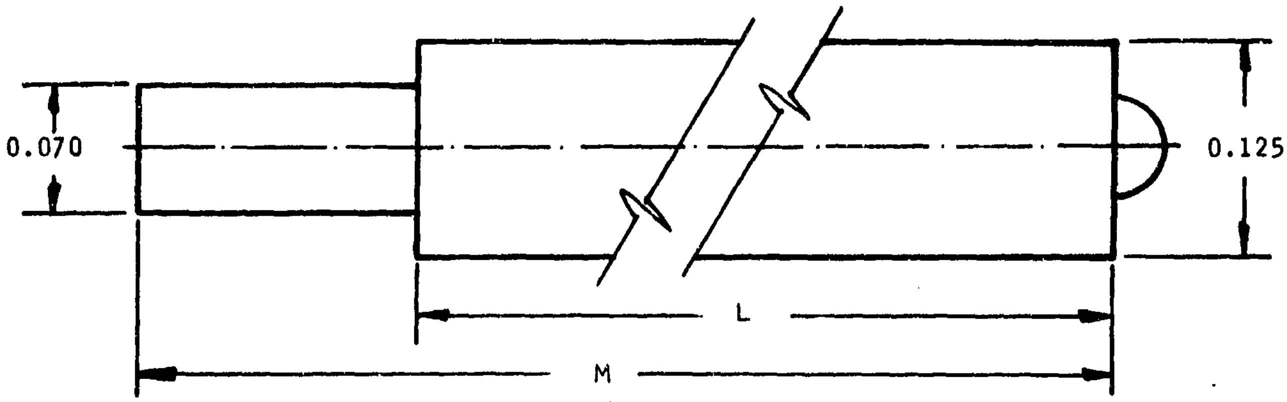
Group III-3 Firing Pins



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
60	5.5556×10^{-6}	0.973	0.885	1.197

Figure D-7a

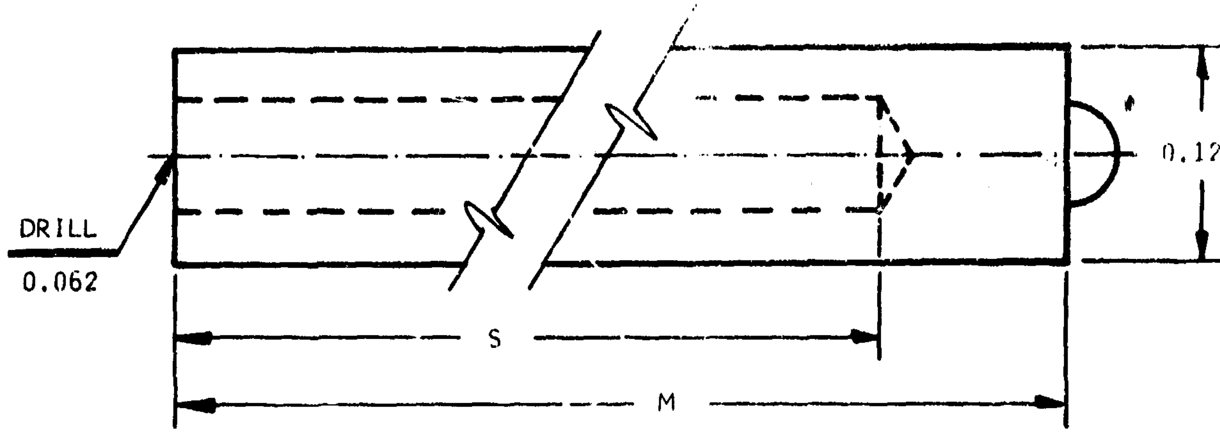
Group IV-1 Firing Pin



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
45	5.1020×10^{-6}	0.893	0.548	0.860
51	4.6875	0.821	0.502	0.814
52	3.9062	0.684	0.415	0.727
56	5.000	0.876	0.537	0.849
57	4.000	0.700	0.425	0.737
58	3.000	0.525	0.314	0.626
59	2.500	0.438	0.258	0.570
61	4.8611	0.851	0.521	0.833
62	4.1666	0.730	0.444	0.756
63	3.4722	0.608	0.367	0.679
64	2.7778	0.486	0.289	0.601
67	4.0816	0.715	0.434	0.746
68	3.5714	0.625	0.378	0.690
69	3.0612	0.536	0.321	0.633
70	2.5510	0.447	0.264	0.576
74	3.125	0.547	0.328	0.640
75	2.7343	0.479	0.285	0.597

Figure D-7b

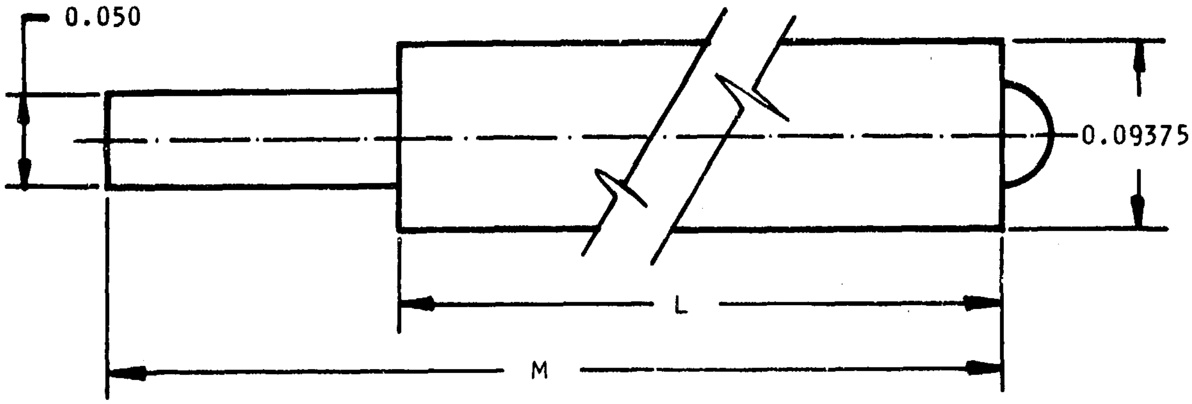
Group IV-2 Firing Pins



I.D. No.	Mass		M	S
	lb-sec²/in	gram		
65	2.0833×10^{-6}	0.365	0.601	0.541
66	1.7361	0.304	0.549	0.489
71	2.0408	0.357	0.595	0.535
72	1.5306	0.268	0.519	0.459
73	1.2755	0.223	0.481	0.421
76	2.3437	0.410	0.639	0.579
77	1.9531	0.342	0.582	0.522
78	1.5625	0.274	0.524	0.464
81	2.4691	0.432	0.658	0.598
82	2.1604	0.378	0.612	0.562
83	1.8518	0.324	0.567	0.507
84	1.5432	0.270	0.521	0.461
88	2.0000	0.350	0.588	0.528
89	1.7500	0.306	0.551	0.491
90	1.5000	0.263	0.514	0.454

Figure D-7c

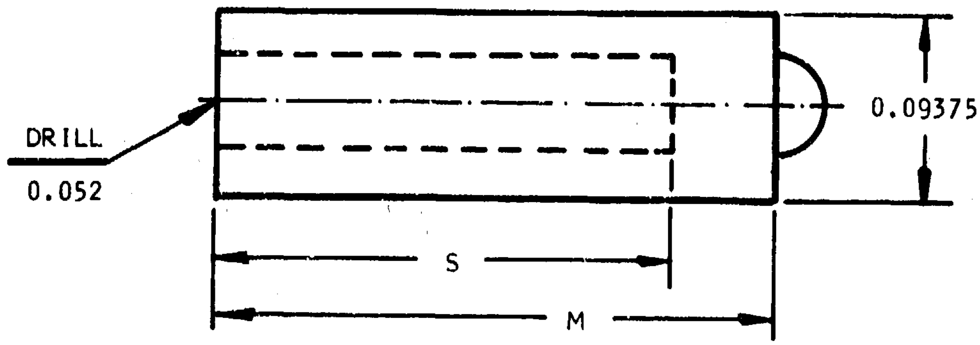
Group IV-3 Firing Pins



I.D. No.	Mass		L	M
	lb-sec^2/in	gram		
79	1.1718×10^{-6}	0.205	0.204	0.391
80	0.9765	0.171	0.165	0.352
85	1.2345	0.216	0.216	0.403
86	0.9259	0.162	0.155	0.342
91	1.250	0.219	0.219	0.406
92	1.000	0.175	0.170	0.357
95	1.6528	0.289	0.299	0.487
96	1.4462	0.253	0.258	0.445
97	1.2396	0.217	0.217	0.404
98	1.0330	0.181	0.176	0.363
99	0.8264	0.145	0.136	0.323
102	1.3888	0.243	0.247	0.434
103	1.2152	0.213	0.212	0.399
104	1.0416	0.182	0.178	0.365
105	0.8680	0.152	0.143	0.330

Figure D-8a

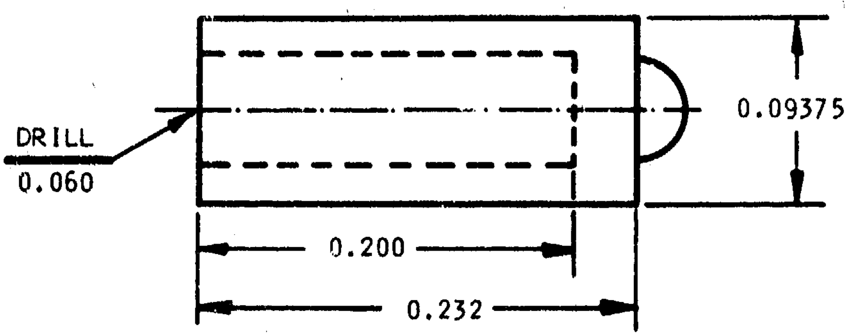
Group V-1 Firing Pins



I.D. No.	Mass		M	S
	lb-sec^2/in	gram		
87	0.7716×10^{-6}	0.1351	0.317	0.287
93	0.750	0.1313	0.311	0.281
94	0.625	0.1094	0.275	0.245
100	0.6198	0.1085	0.274	0.244
101	0.5165	0.0904	0.244	0.214
106	0.6944	0.1216	0.295	0.265
107	0.5208	0.0912	0.245	0.215

Figure D-8b

Group V-2 Firing Pins



I.D. No.	Mass	
	lb-sec²/in	gram
108	0.4340x10^{-6}	0.0760

Figure D-8c

Group V-3 Firing Pin

b. Selection of Springs

Figure D-9 lists the springs which were used with the various groups of firing pins.

Group	I	II	III	IV	V
Lee Cat. Number	LC-063G-12	LC-045D-18	LC-024B-15	Special	Special
Wire Dia. (in)	0.063	0.045	0.024	0.022	0.018
Outside Diameter (in)	0.470	0.300	0.180	0.117	0.088
Free Length (in)	2.500	2.250	2.000	1.500	1.625
Active Turns	16 approx.	22 approx.	30 approx.	31 approx.	48 approx.
Load at Solid Ht. (lbs)	29	19.75	5.4	8.5	6.5
Mass of Spring (grams)	8.988	3.888	0.903	0.476	0.355

Figure D-9

Springs Used for Various Firing Pin Groups

The following computation serves to illustrate the manner in which the springs were selected. In addition it shows that a velocity of 1200 in/sec represents the upper limit of the possible velocities for the types of mass spring systems involved.

Given: A spring for Group V firing pins:

Outside diameter	=	0.088 in.
Wire diameter (d)	=	0.018 in.
Mean coil diameter (D)	=	0.070 in.
Number of active coils	=	48
Free length	=	1.625 in.
Spring mass (m_s)	=	0.355 grams
Spring material	=	music wire

Firing pin corresponding to identification number 102, i.e.

$$M = 1.3888 \times 10^{-6} \text{ (lb-sec}^2\text{/in)}$$
$$= 0.243 \text{ (grams)}$$

Required firing pin velocity for identification number 102:

$$v = 1200 \text{ (in/sec)}$$

Find: Is it possible for the present system to impart a velocity of 1200 in/sec to the firing pin?

To obtain the answer to the above question it is first necessary to determine the uncorrected shear stress which the spring must experience in order to produce the prescribed firing pin velocity. After that it must be determined whether the necessary spring deflection can be obtained with the existing solid height of the spring.

Finally the curvature correction factor must be determined for the spring and with it one may obtain the corrected shear stress. If the resulting shear stress is much beyond 200,000 psi it is likely that the spring will take a set and cannot properly perform.

Equation (B-85) together with Eq.(A-49) gives the maximum velocity attained by a spring driven mass:

$$v_{max} = \frac{\tau_f}{131}\left(-1 + 2e^{-\frac{e}{2}^{-\frac{2m_S}{M}}}\right) \tag{D-3}$$

This holds because the spring is made of steel. For a mass ratio M/m_s = 0.243/0.355 = 0.685 the factor in parenthesis becomes according to Figure B-7 approximately 0.943. Substituting this factor as well as the desired velocity of 1200 in/sec in the above one may solve for the required uncorrected shear stress:

$$\tau_f = \frac{1200 \times 131}{0.943} = 166{,}700 \text{ psi} \tag{D-4}$$

The required deflection of the spring is determined with the help of Equations (A-46) and (A-1a):

$$f = \frac{\tau_f \pi D^2 N}{Gd} \qquad \text{(D-5)}$$

With G = 11.5 x 10^{-6} and the above τ_f one obtains:

$$f = 0.595 \qquad \text{(D-6)}$$

Since the free length equals 1.625 inches and the solid height equals (48 + 2) x 0.018 = 0.900, the spring can be deflected a total of 0.725 inches. Therefore f = 0.595 is well within the range and the free length need not be changed.

So far it would seem that the spring is perfectly satisfactory. Now consider that the spring index

$$c = \frac{D}{d} = \frac{0.070}{0.018} = 3.888 \qquad \text{(D-7)}$$

and that therefore the curvature correction factor

$$K = \frac{4c - 1}{4c - 4} + \frac{0.615}{c} = 1.417 \qquad \text{(D-8)}$$

The corrected shear stress now becomes:

$$\tau_{cor} = \tau_f K = 236{,}200 \text{ psi}$$

The above becomes marginal when one considers that the shear stress for music wire should not exceed 200,000 psi by too much.

This high stress means that the spring will take a certain set when assembled the first time and strain hardening will take place. While this raises the elastic limit somewhat and allows the newly formed spring to do its job if the free length remains sufficient, it is not at all desirable when the spring is to be repeatedly used.

There was a certain amount of trouble with this spring during the performance of the experiment. It was overcome by such brute force means as stretching the spring repeatedly.

Since all efforts at designing a better spring failed, it can be safely stated that 1200 in/sec does not represent a practically attainable firing pin speed. (This design difficulty had its reason in the fact that a single spring type had to accomodate many firing pins.)

c. Determination of Firing Pin Velocity

The following describes the manner in which the velocity of the firing pin is measured. Single as well as two probe methods are involved.

1. Single Probe Method

Figure D-10 shows the essentials of the single probe method of measuring the firing pin velocity. The firing pin consists of two reflective surfaces (polished steel) and one nonreflective surface between them. This surface was painted with optical black. As the reflective surface no.1 passes the probe

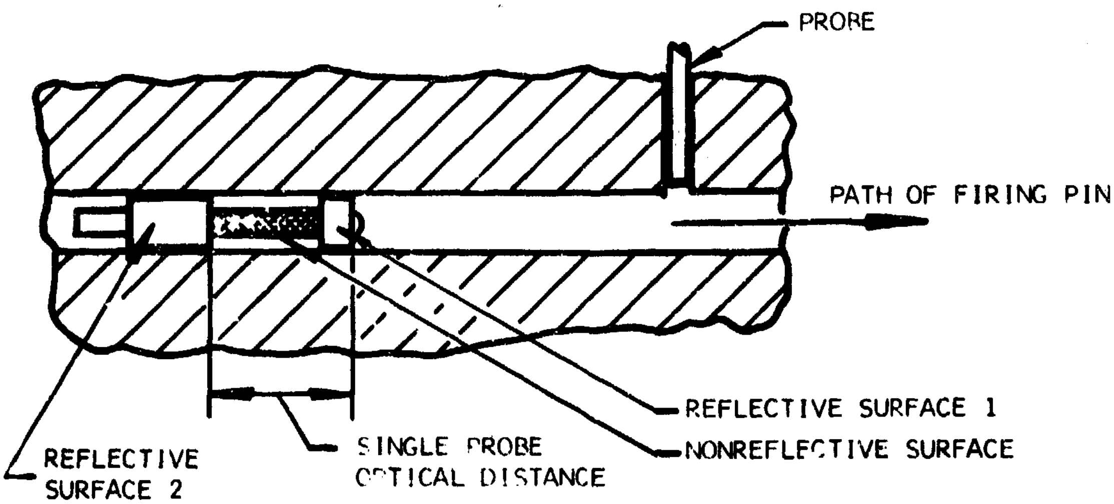


Figure D-10

Single Probe Method of Measuring Velocity

the output voltage of the instrument rises a predetermined amount and triggers the counter. As the nonreflective surface passes the probe the voltage decreases, while the counter continues to count. As reflective surface no.2 passes the probe, the voltage rises again, and is used to shut off the counter.

The so called "optical distance" between the two points which cause sufficient voltage to switch the counter is determined with the help of a specially modified micrometer. It has been found that the optical distance is usually within less than one percent of the actual distance between the steps.

The velocity of the firing pin is determined by the time t which passes as the firing pin travels through this single probe optical distance L_{op}:

$$V = \frac{L_{op}}{t} \tag{D-9}$$

II.Two Probe Method

The two probe method of measuring the firing pin velocity is applicable to the shorter firing pins, where a reasonably long optical distance of the type illustrated in Figure D-10 cannot be realized on the pin itself. (Conversely, to use the two probe method on the longer firing pins would require too long an optical distance.) Figure D-11 gives a schematic of the two probe method.

The firing pin is reflective throughout its length. As it passes probe no.1 the output voltage rises and starts the counter. Once the pin has passed probe no.1 the voltage drops but the counter continues. When the pin passes probe no.2 the rising voltage shuts off the counter. Again the optical distance must be established by a mechanical measurement of the distance of motion required to operate the counter. This distance conforms closely to the center distance between the probes. Equation (D-9) is also applicable.

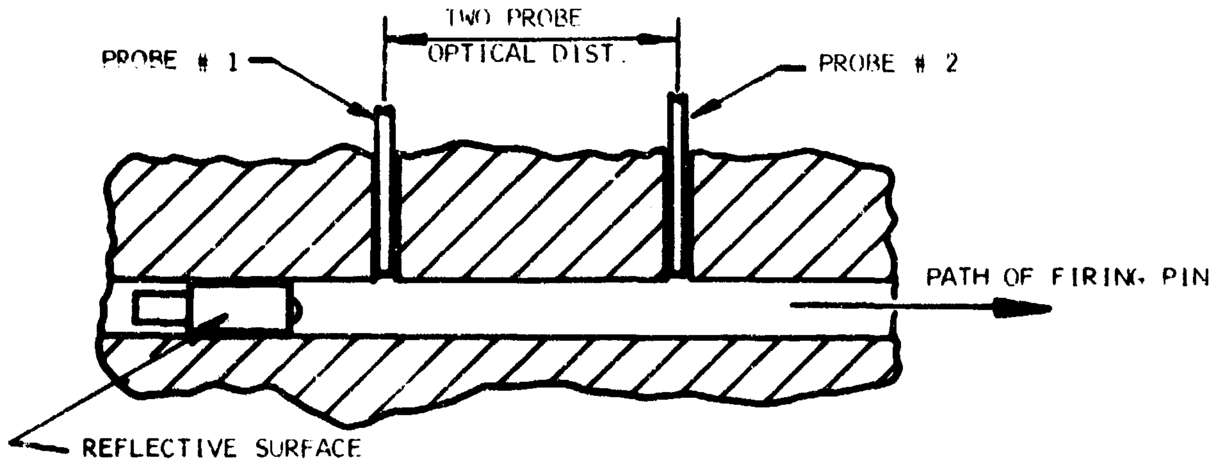


Figure D-11

Two Probe Method of Measuring Velocity

In both methods the optical distance is of the order of 0.750 inches, and the velocity of the firing pin is measured approximately 1.0 inch from the point of impact. This general method of measuring the velocity of the firing pin has been checked independently with a peripherally slotted disc rotating at various known speeds. Excellent correlation was attained (±1 percent).

d. Test Procedure

The tests were run both at an ambient temperature of approximately 70 to 75 degrees F and at -40 degrees F. In order to make sure that handling would not warm up the primers, the cartridges were mounted in the cartridge holders and cooled together to -45 degrees F. The number of firings per energy level, velocity and temperature were planned to be 30. In a few cases there were only 25 firings per identification number, while when there was any doubt concerning the result considerably more firings were made. Figures D-12a and D-12b show the number of test firings for ambient temperature and -40 degrees respectively.

Figure D-13 gives a typical data sheet from an actual run. This test covers I.D. No.60. It represents an energy level of 16 inch-ounces at a velocity of 600 in/sec at -40 degrees F. The required mass of the firing pin is 5.555×10^{-6} lb-sec^2/in, while the actual mass is 5.567×10^{-6} lb-sec^2/in. The average of the optical distance is obtained from four readings as 0.75925 inches. (The firing pin is rotated and readings are taken 90 degrees apart.) The required time for the velocity of 600 in/sec is computed to be 0.001265 seconds.

The 30 data sets record the time indicated by the electronic counter, the spring compression and whether the primer fired.

Ident. No.	Number of Firings	Ident. No.	Number of Firings	Ident. No.	Number of Firings
1	30	37	60	73	--
2	30	38	--	74	30
3	30	39	30	75	30
4	30	40	30	76	30
5	57	41	30	77	30
6	120	42	30	78	30
7	118	43	120	79	30
8	240	44	30	80	30
9	85	45	--	81	30
10	90	46	30	82	30
11	90	47	30	83	30
12	120	48	30	84	60
13	119	49	30	85	30
14	210	50	120	86	30
15	360	51	30	87	30
16	330	52	--	88	30
17	480	53	30	89	30
18	30	54	30	90	60
19	30	55	30	91	30
20	60	56	30	92	30
21	30	57	30	93	30
22	240	58	30	94	30
23	120	59	--	95	30
24	90	60	30	96	30
25	30	61	30	97	30
26	30	62	30	98	30
27	30	63	30	99	30
28	30	64	30	100	30
29	120	65	30	101	--
30	60	66	--	102	30
31	30	67	30	103	30
32	30	68	30	104	30
33	30	69	30	105	30
34	30	70	30	106	30
35	30	71	30	107	30
36	30	72	30	108	--

Figure D-12a

Number of Test Firings at Ambient Temperature

Ident. No.	Number of Firings	Ident. No.	Number of Firings	Ident. No.	Number of Firings
1	25	37	30	73	30
2	25	38	--	74	30
3	25	39	30	75	30
4	55	40	30	76	30
5	55	41	30	77	30
6	60	42	30	78	30
7	90	43	120	79	30
8	60	44	30	80	--
9	60	45	--	81	30
10	30	46	30	82	30
11	30	47	30	83	30
12	30	48	30	84	30
13	210	49	30	85	30
14	120	50	120	86	90
15	150	51	30	87	--
16	270	52	--	88	30
17	30	53	30	89	30
18	30	54	30	90	30
19	30	55	30	91	30
20	150	56	30	92	30
21	150	57	30	93	30
22	60	58	30	94	--
23	150	59	--	95	30
24	120	60	30	96	30
25	30	61	30	97	30
26	90	62	30	98	30
27	90	63	30	99	30
28	30	64	30	100	30
29	150	65	30	101	--
30	30	66	--	102	30
31	--	67	30	103	30
32	90	68	30	104	30
33	30	69	30	105	30
34	30	70	30	106	30
35	60	71	30	107	30
36	90	72	30	108	--

Figure D-12b

Number of Test Firings at -40 Degrees F

Throughout the tests it was made a rule to count only those firings where the actual time did not deviate more than ±2.5 percent from the computed time. (The stars on the data sheet indicate that the tube was cleaned preceding the particular firing.)

The 5 in-oz tests were discontinued as it became apparent that it was impossible to attain reproducible results.

4. Test Results

a. Results of Firing Tests

Figure D-14 is a tabulation of the results of all the firing tests at both ambient temperature and at -40 degrees F. It shows the percentage of primers fired at the various energy levels and firing pin velocities. As was stated earlier, the 5 in-oz tests were discontinued since it was not possible to obtain reproducible results.

Figures D-15a and D-15b presents the identical data in a graphical form similar to the "Erwood Curves". Figure D-15a gives the results for ambient temperature while Figure D-15b deals with -40 degrees F.

b. Verification of the Theory Concerning Velocities of Spring Driven Masses

The spring compression associated with each test velocity was recorded for all data points. (See sample data sheet in Figure D-13.) This makes it possible to compare the measured nominal velocities (v_N) with the theoretical velocities (v_T) which correspond to the actual spring compressions.

This procedure, which allows the determination of a design correction factor, will now be outlined by way of an example.

Figure D-13 gives the following test results for I.D. 60-B (-40°F):

Firing pin velocity:	600 in/sec
Firing pin mass:	0.975 grams
Spring compression (f):	0.445 inches

This spring belongs to Group IV (see Figure D-9), and has the following dimensions:

Outside diameter:	0.120 inches
Wire diameter (d):	0.022 inches
Mean coil diameter (D):	0.095 inches
Number of active turns (N):	31
Mass of spring (m_s):	0.476 grams

The velocity v_T which corresponds theoretically to the above spring compression f is now computed with the help of Eq.(D-3):

$$v_T = v_{max} = \frac{\tau_f}{131}\left(-1 + 2e^{-\frac{e}{2}^{\frac{-2m_s}{M}}}\right) \qquad \text{(D-10)}$$

The shear stress associated with the deflection f is given by rearrangement of Eq.(D-5):

FIRING TEST NO. I D # 66 - 13

ROOM TEMP. (F): ________ DATE: 8/19/69

COLD TEST TEMP. (F): -40° F TESTER: GR & KF

NOMINAL ENERGY (IN-OZ) 16 NOMINAL VELOCITY (IN/SEC) 600

NOM. MASS (#-SEC^2/IN) 5555 x10^{-6} ACTUAL MASS (#-SEC^2/IN) 5.567 x10^{-6}

ACTUAL MASS (g) .975

OPTICAL DISTANCE (IN) (L_1=.764 L_2=.757 L_3=.757 L_4=.759 $\overline{L}$ .75925)
(TAKEN 90° APART)

NOMINAL TIME = $\overline{L}/V_{nom}$ = .0012654 (SEC)

SHOT NO.	TIME (SEC)	SPRING COMP. (IN)	F	NF	SHOT NO.	TIME (SEC)	SPRING COMP. (IN)	F	NF
* 1	.001290	.445	✓		16	.001248	.445	✓	
2	.001272	↓	✓		* 17	.001242	↓	✓	
* 3	.001279	↓	✓		18	.001251	↓	✓	
4	.001290	↓	✓		* 19	.001261	↓	✓	
* 5	.001240	↓	✓		20	.001263	↓	✓	
6	.001268	↓	✓		* 21	.001254	↓	✓	
* 7	.001271	↓	✓		22	.001291	↓	✓	
8	.001247	↓	✓		* 23	.001273	↓	✓	
* 9	.001247	↓	✓		24	.001286	↓	✓	
10	.001286	↓	✓		* 25	.001243	↓	✓	
* 11	.001290	↓	✓		26	.001272	↓	✓	
12	.001287	↓	✓		* 27	.001279	↓	✓	
* 13	.001253	↓	✓		28	.001260	↓	✓	
14	.001289	↓	✓		* 29	.001253	↓	✓	
* 15	.001291	↓			30	.001269	↓	✓	

F-FIRED
NF-NOT FIRED

PERCENT FIRED: 30K - 150

Figure D-13
Typical Data Sheet

VELOCITY OF FIRING PIN (IN/SEC)	16 In-Oz		14 In-Oz		12 In-Oz		10 In-Oz		8 In-Oz		6 In-Oz		5 In-Oz	
	AMB.	-40^oF	AMB.	-40^oF	AMB.	-40^oF	AMB.	-40^oF	AMB.	-40^oF	AMB.	-40^oF	AMB.	-40^oF
50					23.3	8.0	10.0	4.0	13.3	4.0				
100	100.0	100.0	100.0	85.5	98.9	95.0	86.4	62.0	75.8	61.7	58.8	33.3	31.1	10.0
150	100.0	100.0	100.0	100.0	99.0	94.0	94.8	85.0	88.0	69.3	61.4	45.5	40.6	46.7
200	100.0	100.0	100.0	100.0	100.0	95.8	100.0	91.1	92.8	88.3	82.2	58.0	54.7	45.0
250	100.0	100.0	100.0	100.0	100.0	98.9	100.0	100.0	98.0	92.5	83.3	73.3	70.0	-
300	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	90.0	86.6	-	-
350	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	98.3	98.3	96.6	96.6	-	-
400	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	98.3	99.2	93.3	96.6	-	-
500	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	96.6	93.3	-	-
600	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	-
700	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	96.6
800	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	93.3	-
900	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	90.0	-
1000	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-
1100	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	-
1200	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	-	90.0

Figure D-14

M 42 G Primer Firing Data

(Percentage of primers fired at various energy levels, firing pin velocities, and temperatures.)

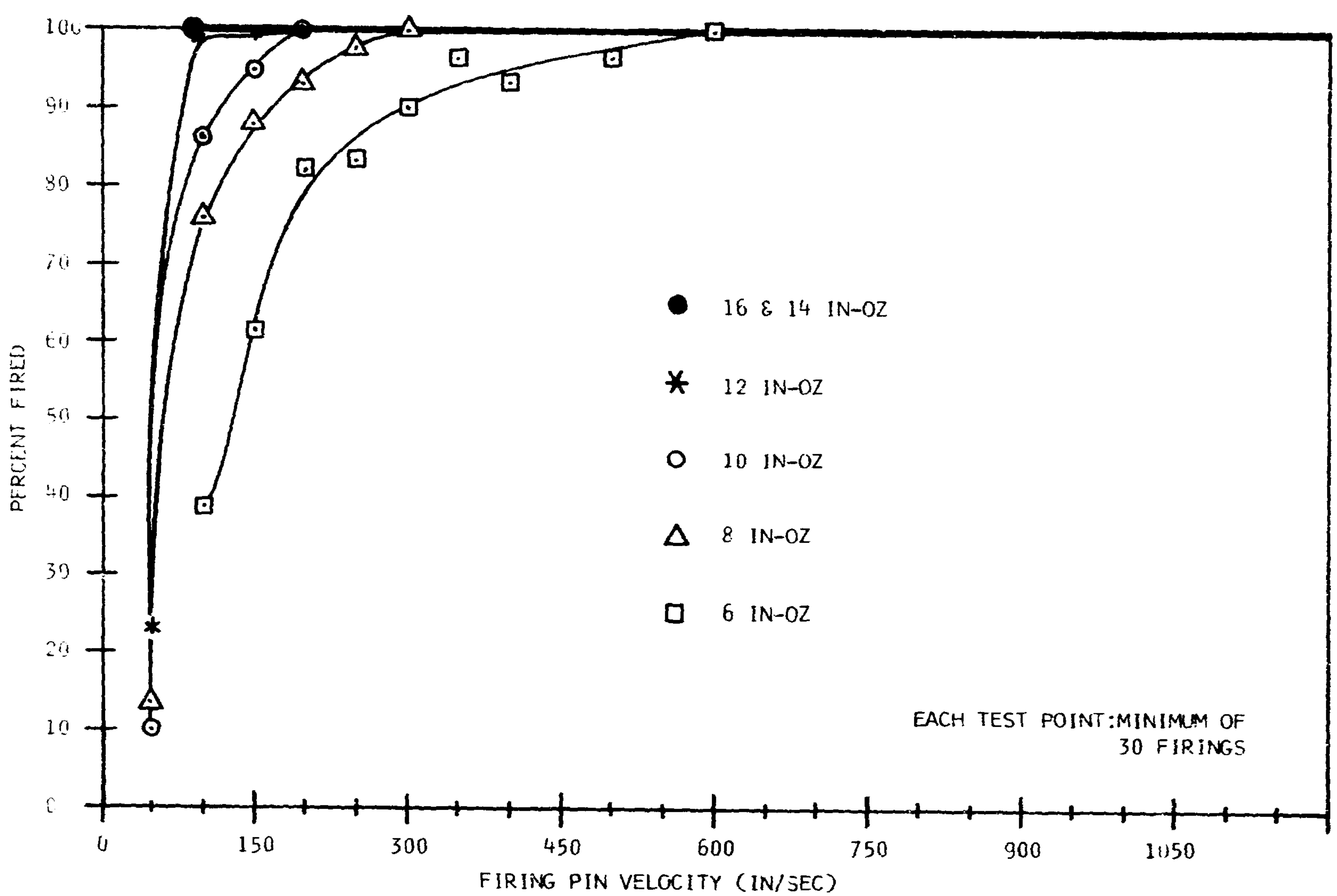


Figure D-15a:

M42G Primer Firing Curves (Ambient Temperature)

(C.C.N.Y. Tests)

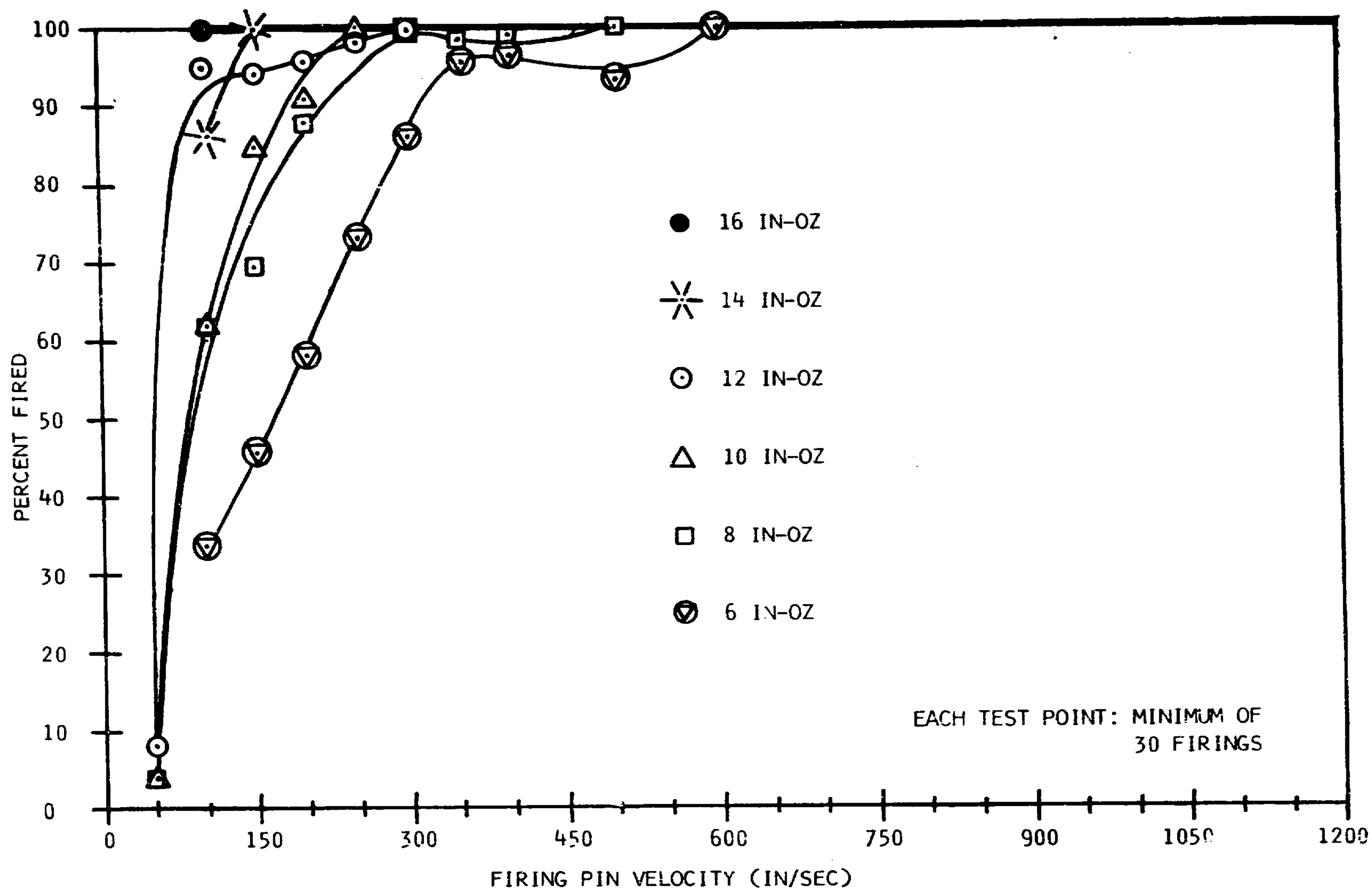


Figure D-15b:

M42G Primer Firing Curves (-40 deg. F.)

(C.C.N.Y. Tests)

$$\tau_f = \frac{Gdf}{\pi D^2 N} = \frac{(11.5 \times 10^6)(0.022)(0.445)}{(3.1416)(0.095)^2(31)} = 128{,}500 \text{ psi} \tag{D-11}$$

With

$$\frac{M}{m_S} = 2.05$$

the factor in parenthesis of Eq.(D-10) becomes according to Figure B-7 approximately 0.656.

Substitution of the above into Eq.(D-10) leads to:

$$v_T = \frac{128{,}500}{131} \times 0.656 = 643.6 \text{ in/sec.}$$

This is larger than the actual velocity of 600 in/sec. The percentage increase with respect to the nominal velocity is given by:

$$\text{percentage increase} = \frac{v_T - v_N}{v_N} \times 100 = \frac{643.6 - 600}{600} \times 100$$

$$= 7.26 \text{ percent}$$

This means that in order to actually attain a certain nominal velocity (v_N), the spring deflection must be computed for a theoretical velocity (v_T) which is higher than the nominal one by a certain percentage. Thus:

$$v_T = v_N(1 + C_D) \tag{D-12}$$

where the design correction factor is given by:

$$C_D = \frac{v_T - v_N}{v_N} \tag{D-13}$$

Figure D-16 gives a tabulation of the correction factors C_D for all test runs. Except for I.D. numbers 4 and 5, and whenever the spring took a set, i.e. the spring characteristics changed, the value of the correction factor does not exceed 0.25. In the case of I.D.'s 4 and 5 it was found that high friction between the firing pin and the tube was responsible for the need of increased spring compression to attain nominal velocity. (The use of a Teflon spray proved very helpful in reducing friction.) It needs to be pointed out that in the experimental setup the firing pin traveled as far as 4 inches after separation from the spring in order to facilitate the velocity measurement. In an actual situation the firing pin travel would be much shorter and thus the possibility for speed reducing friction work would be very much smaller.

The spring set can definitely be avoided by limiting the corrected shear stress in the spring to approximately 200,000 psi.

Lastly, the presence of the spring pad (see Figure D-3) contributed to the slowing down of the firing pin.

For design purposes one may safely assume the general correction factor

$$C_D = 0.25 \tag{D-14}$$

Appendix E, which deals with the space optimization of spring driven primer striker systems makes use of the above general correction factor.

Identification Number	Group	Kinetic Energy (In-Oz)	Mass of Firing Pin $(\frac{Lb\text{-}Sec^2}{In}) \times 10^6$	Nominal Velocity (v_N) (In/Sec)	Correction Factor $C_D = \frac{v_T - v_N}{v_N}$ Ambient Test	- 40°F Test
1	I	12	600	50	*	*
2	I	10	500	50	*	*
3	I	8	400	50	*	*
4	I	16	200	100	0.28	0.27
5	I	14	175	100	0.33	0.26
6	II	12	150	100	*	*
7	II	10	125	100	*	*
8	II	8	100	100	0.16	0.11
9	II	6	75	100	0.15	0.20
10	II	5	62.5	100	0.19	0.23
11	II	16	88.8889	150	0.12	0.11
12	II	14	77.7778	150	0.11	0.14
13	II	12	66.6667	150	0.11	0.14
14	II	10	55.5556	150	0.07	0.11
15	II	8	44.4445	150	0.08	0.15
16	II	6	33.3334	150	0.17	0.16
17	II	5	27.7778	150	0.15	0.14
18	II	16	50.0	200	0.10	0.07

* Not Applicable.

Figure D-16

Correction Factor C_D

Identification Number	Group	Kinetic Energy (In-Oz)	Mass of Firing Pin $(\frac{Lb\text{-}Sec^2}{In}) \times 10^6$	Nominal Velocity (v_N) (In/Sec)	Correction Factor $C_D = \frac{v_T - v_N}{v_N}$	
					Ambient Test	-40° F Test
19	II	14	43.75	200	0.16	0.16
20	II	12	37.50	200	0.09	0.12
21	II	10	31.25	200	0.11	0.11
22	II	8	25.00	200	0.13	0.09
23	II	6	18.75	200	0.11	0.14
24	II	5	15.625	200	0.13	0.16
25	II	16	32.00	250	0.22	0.11
26	II	14	28.00	250	0.18	0.18
27	II	12	24.00	250	0.19	0.15
28	II	10	20.00	250	0.19	0.07
29	II	8	16.00	250	0.12	0.11
30	III	6	12.00	250	0.16	0.06
31	III	5	10.00	250	0.15	-
32	II	16	22.2223	300	0.18	0.15
33	II	14	19.4444	300	0.25	0.22
34	II	12	16.6667	300	0.06	0.13
35	III	10	13.8889	300	0.14	0.08
36	III	8	11.1112	300	0.13	0.13

-: Not Tested.

Figure D-16 (cont.)

Correction Factor C_D

Identification Number	Group	Kinetic Energy (In-Oz)	Mass of Firing Pin $(\frac{Lb\text{-}Sec^2}{In})\times10^6$	Nominal Velocity (v_N) (In/Sec)	Correction Factor $C_D = \frac{v_T - v_N}{v_N}$ Ambient Test	- 40°F Test
37	III	6	8.3334	300	0.14	0.17
38	III	5	6.9444	300	-	-
39	II	16	16.3265	350	0.19	0.16
40	III	14	14.285	350	0.11	0.12
41	III	12	12.2448	350	0.12	0.12
42	III	10	10.204	350	0.10	0.09
43	III	8	8.1632	350	0.13	0.13
44	III	6	6.1224	350	0.15	0.15
45	IV	5	5.102	350	-	-
46	III	16	12.50	400	0.08	0.10
47	III	14	10.9375	400	0.10	0.10
48	III	12	9.375	400	0.15	0.14
49	III	10	7.8125	400	0.11	0.13
50	III	8	6.25	400	0.11	0.12
51	IV	6	4.6875	400	0.22	0.20
52	IV	5	3.9062	400	-	-
53	III	16	8.0	500	0.15	0.11
54	III	14	7.0	500	0.13	0.15

- : Not Tested

Figure D-16 (cont.)

Correction Factor C_D

Identification Number	Group	Kinetic Energy (In-Oz)	Mass of Firing Pin $(\frac{Lb-Sec^2}{In})x10^6$	Nominal Velocity (v_N) (In/Sec)	Correction Factor $C_D = \frac{v_T - v_N}{v_N}$	
					Ambient Test	-40^oF Test
55	III	12	6.0	500	0.14	0.19
56	IV	10	5.0	500	0.13	0.08
57	IV	8	4.0	500	0.14	0.04
58	IV	6	3.0	500	0.22	0.21
59	IV	5	2.5	500	-	-
60	IV	16	5.5555	600	0.07	0.07
61	IV	14	4.8611	600	0.09	0.09
62	IV	12	4.1666	600	0.15	0.08
63	IV	10	3.4722	600	0.17	0.07
64	IV	8	2.7777	600	0.20	0.07
65	IV	6	2.0833	600	0.14	0.19
66	IV	5	1.7361	600	-	-
67	IV	16	4.0816	700	0.22	0.11
68	IV	14	3.5714	700	0.22	0.11
69	IV	12	3.0612	700	**	0.12
70	IV	10	2.5510	700	0.01	0.09
71	IV	8	2.0408	700	**	0.10
72	IV	6	1.5306	700	0.18	0.19

**: Spring Characteristic Has Changed

- : Not Tested

Figure D-16 (cont.)

Correction Factor C_D

Identification Number	Group	Kinetic Energy (In-Oz)	Mass of Firing Pin $\left(\frac{Lb\text{-}Sec^2}{In}\right) \times 10^6$	Nominal Velocity (v_N) (In/Sec)	Correction Factor $C_D = \frac{v_T - v_N}{v_N}$	
					Ambient Test	- 40°F Test
73	IV	[illegible]	1.2755	700	-	0.04
74	IV	[illegible]	3.125	800	0.08	0.14
75	IV	[illegible]	2.7343	800	0.07	0.15
76	IV	12	2.3437	800	0.15	0.03
77	IV	10	1.9531	800	0.16	0.07
78	IV	8	1.5625	800	0.22	0.09
79	V	6	1.1718	800	**	0.24
80	V	3	0.9765	800	**	-
81	IV	16	2.4691	900	0.25	0.09
82	IV	14	2.1604	900	0.20	0.08
83	IV	12	1.8518	900	0.22	0.08
84	IV	[illegible]	1.5432	900	0.09	0.20
85	[illegible]	8	1.2345	900	0.07	0.20
86	V	6	0.9259	900	**	**
87	V	3	[illegible].7716	900	**	-
88	IV	16	[illegible].00	1000	0.12	0.11
89	IV	14	1.75	1000	0.06	0.09
90	IV	12	1.[illegible]9	1000	0.05	0.19

**: Spring Characteristic Has Changed
- Not Tested

Figure D-16 (cont.)

Correction Factor C_D

Identification Number	Group	Kinetic Energy (In-Oz)	Mass of Firing Pin $\left(\frac{Lb\text{-}Sec^2}{In}\right) x10^6$	Nominal Velocity (V_N) (In/Sec)	Correction Factor $C_D = \frac{V_T - V_N}{V_N}$ Ambient Test	Correction Factor $C_D = \frac{V_T - V_N}{V_N}$ - 40°F Test
91	V	10	1.25	1000	0.16	**
92	V	8	1.00	1000	0.24	0.17
93	V	6	0.75	1000	0.24	0.23
94	V	5	0.625	1000	0.24	-
95	V	16	1.6528	1100	0.17	0.22
96	V	14	1.4462	1100	0.19	**
97	V	12	1.2396	1100	0.13	0.18
98	V	10	1.0330	1100	0.17	0.15
99	V	8	0.8264	1100	0.12	0.14
100	V	6	0.6198	1100	**	0.17
101	V	5	0.5165	1100	-	-
102	V	16	1.3888	1200	0.11	0.20
103	V	14	1.2152	1200	0.09	0.17
104	V	12	1.0416	1200	0.04	0.22
105	V	10	0.8680	1200	0.07	**
106	V	8	0.6944	1200	**	**
107	V	6	0.5208	1200	0.25	0.06
108	V	5	0.4340	1200	-	-

**: Spring Characteristic Has Changed
- : Not Tested

Figure D-16 (cont.)

Correction Factor C_D

APPENDIX E

SPACE OPTIMIZATION

The following appendix gives the derivation of the various design equations and outlines the combination of experimental and theoretical results which leads to the final optimization methods.

The resulting optimum design tables, which are listed in Appendix F, allow the design of 100 percent successful spring striker systems for the M42G primer with minimum space needs.

1. Free Length of Optimum Spring

The free length of the optimum spring is obtained with the assumption that this spring is deflected to its solid height and that the spring reaches at solid height its maximum allowable corrected shear stress.

The deflection (f) may then be defined by:

$$f = L_0 - \text{solid height} \qquad \text{(E-1)}$$

where L_0 is the free length of the spring.

When one defines solid height as the product of the wire diameter (d) and the sum of the active turns (N) and two extra turns:

$$\text{solid height} = d(N + 2), \qquad \text{(E-2)}$$

then Eq.(E-1) becomes:

$$f = L_0 - d(N + 2) \qquad \text{(E-3)}$$

From the above the free length of the spring is given by:

$$L_0 = f + d(N + 2) \qquad \text{(E-4)}$$

As was stated above, the spring is to be designed in such a manner that it reaches its maximum allowable corrected shear stress at solid height. In addition its mass must be sufficient to produce the desired velocity of its associated firing pin of mass M.

When the deflection f is such that it produces the maximum allowable corrected shear stress τ_c at solid height, then

$$\tau_c = K\tau_f \qquad \text{(E-5)}$$

where

τ_f = uncorrected stress associated with deflection f

$K = \dfrac{4c - 1}{4c - 4} + \dfrac{0.615}{c}$, the curvature correction factor [42]

$c = \dfrac{D}{d}$, the spring index relating mean coil diameter to wire diameter.

Equation (E-5) is now to be expressed in terms of the deflection f. According to Eq.(A-45):

$$\tau_f = \frac{8P_f D}{\pi d^3} ,$$

and according to Eq.(A 1a):

$$P_f = \lambda f = \frac{Gd^4 f}{8D^3 N}$$

Thus Eq.(E-5) becomes:

$$\tau_c = f\frac{KGd}{ND^2\pi}$$

And finally:

$$f = \frac{\tau_c N\pi D^2}{KGd} \tag{E-6}$$

The mass of the spring, in the sense of Appendix B, is due to the active coils only:

$$m_s = N(\pi D)(\frac{\pi d^2}{4})\frac{\gamma}{g} \tag{E-7}$$

and the number of active turns may then be expressed by:

$$N = \frac{4m_s g}{\pi^2 \gamma D d^2} \tag{E-8}$$

Substitution of Eq's.(E-6) and (E-8) into Eq.(E-4) gives for the free length of the spring:

$$L_0 = \frac{4m_s g}{\pi^2 \gamma}(\frac{\tau_c \pi D}{KGd^3} + \frac{1}{Dd}) + 2d \tag{E-9}$$

When the above is expressed in terms of the spring index c, it becomes:

$$L_0 = \frac{4m_s gc^2}{\pi^2 \gamma D^2}(\frac{\tau_c \pi c}{KG} + \frac{1}{c}) + 2\frac{D}{c} \tag{E-10}$$

2. Optimum Spring Mass m_s

The optimum spring mass is now obtained with the help of the maximum velocity equations (D-10) or (B-85):

$$v_T = \frac{\tau_f}{131}\left(-1 + 2e^{-\frac{e^{-\frac{2m_s}{M}}}{2}}\right) \tag{E-11}$$

Of course the theoretical velocity v_T must be attained without exceeding the corrected shear stress τ_c at solid height. In addition, the theoretical velocity(v_T) must be expressed in terms of the corrected nominal velocity (v_N) according to Eq.(D-12). The latter is necessary since the design procedure is based on the experimentally attained 100 percent firing points as well as on the experience with the springs.

Thus Eq.(E-11) is written in the form:

$$v_N(1 + C_D) = \frac{\tau_c}{131\ K}\left(-1 + 2e^{-\frac{e^{-\frac{2m_s}{M}}}{2}}\right) \tag{E-12}$$

The required spring mass m_s is now obtained from the above:

$$m_s = -\frac{M}{2}\ln\ [-2\ \ln\ B] \tag{E-13}$$

where

$$B = \frac{131\ Kv_N(1 + C_D)}{2\tau_c} + 0.5 \tag{E-14}$$

Equation (E-13) is valid only for certain ranges of the factor B: First, as shown in Appendix B the ratio $M/m_s \leq 5.69$, and in addition, to obtain a positive value for m_s it is necessary that $0.606 < B < 1.000$.

Therefore for the usual value of the corrected maximum shear stress τ_c = 200,000 psi, and the general value $C_D = 0.25$ (see Eq.(D-14)), the nominal velocity which may be used in the above computations is restricted to the following limits:

$$\left[v_{N(min)}\right]K > 260 \text{ in/sec}$$

and

$$\left[v_{N(max)}\right]K < 1220 \text{ in/sec}$$

3. Overall Space Requirement of Optimum System

The overall height L_t of the mass-spring system is now determined as the combined length of the firing pin L_p, the solid height of the spring, and the total distance traveled by the mass M until it reaches the maximum attainable velocity which corresponds to the deflection F. The latter dimension was given as $F_t = fC_f$ in Eq.(C-13) of Appendix C.

With this concept one finds:

$$L_t = fC_f + d(N + 2) + L_p \tag{E-15}$$

Substitution of Eq's.(E-6) and (E-8) leads after rearrangement to:

$$L_t = \frac{4m_s gc^2}{\pi^2 \gamma D^2}\left(\frac{\tau_c \pi c}{KG} C_f + \frac{1}{c}\right) + 2\frac{D}{c} + L_p \tag{E-16}$$

4. Design Equations and Optimization Procedure

The combination of experimental and theoretical results of the present investigation, which was used to devise an optimization procedure, will now be explained in detail.

a. Use of Experimental Data

The design procedure aims to recreate those conditions which led to 100 percent firing of the M42G primers during testing.

To this end the following successful test points (see also Figures D-2 and D-14 in Appendix D) were initially considered:

16 in-oz:	I.D.:	32, 39, 46, 53, 60, 67, 74, 81, 88, 95, 102
14 in-oz:	I.D.:	33, 40, 47, 54, 61, 68, 75, 82, 89, 96, 103
12 in-oz:	I.D.:	41, 48, 55, 62, 69, 76, 83, 90, 97, 104
10 in-oz:	I.D.:	42, 49, 56, 63, 70, 77, 84, 91, 98, 105
8 in-oz:	I.D.:	71, 78, 85, 92, 99, 106
6 in-oz:	I.D.:	79, 86, 93, 100, 107

b. Space Restrictions and Firing Pin Dimensions

In order to stay within reasonable dimensions for the resulting spring driven primer striker systems, it was decided that the diameter of the firing pin (and with that the diameter of the spring) should not exceed 0.375 inches, and that the overall height of the system, as described by Eq.(E-16) should not exceed 2.000 inches.

Figure E-1 gives various possible firing pin lengths which correspond to feasible firing pin diameters for the firing pin masses associated with the I.D.'s considered. (See last section.) The material is assumed to be steel (γ = 0.283 lbs/in^3).
The following diameters are examined for each I.D. number: 0.375, 0.312, 0.250, 0.218, 0.200, 0.187, 0.170, 0.156, 0.140, 0.125, 0.110, 0.100, and 0.093 inches.

Lengths below 0.125 inches were generally excluded.

NOTE: The firing pin is assumed to be a solid cylinder without consideration of the 0.03 inch length of the tip, or any reduced diameter for locating it within the inside diameter of the spring.

IN OZ =16

I.D. NO.	MASS(E6)	.375	.312	.250	.218	.200	.187	.170	.156	.140	.125	.110	.100	.093
32	22.222	0.275	0.395	0.613	0.807	0.965	1.098	1.336	1.583	1.970	2.472	3.192	3.663	4.418
39	16.326	0.202	0.291	0.454	0.593	0.709	0.877	0.982	1.163	1.448	1.816	2.345	2.639	3.246
46	12.500	0.154	0.222	0.348	0.454	0.543	0.618	0.752	0.890	1.108	1.390	1.795	2.173	2.485
53	8.000		0.142	0.222	0.291	0.348	0.395	0.481	0.570	0.709	0.890	1.149	1.391	1.590
60	5.555			0.154	0.202	0.241	0.275	0.334	0.396	0.493	0.618	0.798	0.966	1.104
67	4.081				0.148	0.177	0.202	0.245	0.291	0.362	0.454	0.586	0.709	0.811
74	3.125					0.136	0.154	0.188	0.223	0.277	0.348	0.449	0.543	0.621
81	2.469						0.122	0.148	0.176	0.219	0.275	0.355	0.429	0.491
88	2.000								0.142	0.177	0.222	0.287	0.348	0.398
95	1.652									0.146	0.184	0.237	0.287	0.328
102	1.388									0.123	0.154	0.194	0.241	0.276

IN OZ =14

I.D. NO.	MASS(E6)	.375	.312	.250	.218	.200	.187	.170	.156	.140	.125	.110	.100	.093
33	19.444	0.240	0.346	0.541	0.706	0.845	0.961	1.169	1.385	1.724	2.163	2.793	3.380	3.866
40	14.285	0.177	0.254	0.397	0.519	0.621	0.706	0.859	1.018	1.267	1.589	2.052	2.483	2.840
47	10.938	0.135	0.195	0.304	0.397	0.475	0.541	0.658	0.779	0.970	1.216	1.571	1.901	2.174
54	7.000		0.125	0.195	0.254	0.304	0.346	0.421	0.499	0.621	0.779	1.005	1.217	1.392
61	4.861			0.135	0.177	0.211	0.240	0.292	0.346	0.431	0.541	0.698	0.845	0.966
68	3.571				0.130	0.155	0.177	0.215	0.254	0.317	0.397	0.513	0.621	0.710
75	2.734					0.119	0.135	0.164	0.195	0.242	0.304	0.393	0.475	0.544
82	2.160						0.107	0.130	0.154	0.192	0.240	0.310	0.376	0.429
89	1.750								0.125	0.155	0.195	0.251	0.304	0.348
96	1.446									0.128	0.161	0.208	0.251	0.288
103	1.215										0.135	0.175	0.211	0.242

Figure E-1

Firing Pin Lengths of Initially Considered Test Masses Corresponding to Diameters from 0.375 to 0.093 Inches.

IN OZ =12

I.D. NO.	MASS(E6)	.375	.312	.250	.218	.200	.187	.170	.156	.140	.125	.110	.100	.093
41	12.245	0.151	0.218	0.340	0.445	0.532	0.605	0.736	0.872	1.086	1.362	1.759	2.128	2.434
48	9.375		0.167	0.261	0.340	0.407	0.463	0.564	0.668	0.831	1.043	1.347	1.630	1.864
55	6.000			0.167	0.218	0.261	0.297	0.361	0.427	0.532	0.667	0.862	1.043	1.193
62	4.167				0.151	0.181	0.206	0.251	0.297	0.369	0.463	0.598	0.724	0.828
69	3.061					0.133	0.151	0.184	0.218	0.271	0.340	0.440	0.532	0.609
76	2.344					0.102	0.116	0.141	0.167	0.208	0.261	0.337	0.407	0.466
83	1.852							0.111	0.132	0.164	0.206	0.266	0.322	0.368
90	1.500									0.133	0.167	0.215	0.261	0.299
97	1.240										0.138	0.178	0.215	0.246
104	1.042											0.150	0.181	0.207

IN OZ =10

I.D. NO.	MASS(E6)	.375	.312	.250	.218	.200	.187	.170	.156	.140	.125	.110	.100	.093
42	10.204	0.126	0.182	0.284	0.371	0.443	0.504	0.614	0.727	0.905	1.135	1.456	1.774	2.029
49	7.813		0.139	0.217	0.284	0.339	0.386	0.470	0.556	0.693	0.869	1.172	1.358	1.553
56	5.000			0.139	0.182	0.217	0.247	0.301	0.356	0.443	0.556	0.718	0.869	0.994
63	3.472				0.126	0.151	0.172	0.209	0.247	0.308	0.386	0.499	0.604	0.690
70	2.551					0.111	0.126	0.153	0.182	0.226	0.284	0.366	0.443	0.507
77	1.953						0.097	0.117	0.139	0.173	0.217	0.281	0.339	0.388
84	1.543							0.093	0.110	0.137	0.172	0.222	0.268	0.307
91	1.250										0.139	0.180	0.217	0.249
98	1.033											0.148	0.180	0.205
105	0.868											0.125	0.151	0.173

Figure E-1 (Continued)

IN OZ = 8

I.D. NO.	MASS(E6)	.375	.312	.250	.218	.200	.187	.170	.155	.140	.125	.110	.100	.093
71	2.041						0.101	0.123	0.145	0.161	0.227	0.293	0.355	0.406
78	1.562							0.094	0.111	0.138	0.174	0.224	0.272	0.311
85	1.234								0.088	0.109	0.137	0.177	0.214	0.245
92	1.000										0.111	0.144	0.174	0.199
99	0.826											0.119	0.144	0.164
106	0.694											0.100	0.121	0.138

IN OZ = 6

I.D. NO.	MASS(E6)	.375	.312	.250	.218	.200	.187	.170	.155	.140	.125	.110	.100	.093
72	1.531							0.092	0.109	0.136	0.170	0.220	0.266	0.304
79	1.172								0.083	0.104	0.130	0.168	0.204	0.233
86	0.926									0.082	0.103	0.133	0.161	0.184
93	0.750											0.108	0.130	0.149
100	0.620													0.123
107	0.521													

Figure E-1 (Continued)

c. Design Equations

The design equations used in the following optimization proceedure are now recapitulated:

The free length of the spring is given by Eq.(E-10):

$$L_0 = \frac{4m_s gc^2}{\pi^2 \gamma D^2}\left(\frac{\tau_c \pi c}{KG} + \frac{1}{c}\right) + 2\frac{D}{c} \tag{E-17}$$

where according to Eq.(E-13):

$$m_s = -\frac{M}{2}\ln(-2\ln B) \tag{E-18}$$

and

$$B = \frac{131\ Kv_N(1 + C_D)}{2\tau_c} + 0.5 \tag{E-19}$$

further

$$c = \frac{D}{d} \tag{E-20}$$

and

$$K = \frac{4c - 1}{4c - 4} + \frac{0.615}{c} \tag{E-21}$$

The number of active turns of the spring is given by Eq.(E-8):

$$N = \frac{4m_s g}{\pi^2 \gamma D d^2} \tag{E-22}$$

Finally, the overall height of the system, i.e. the space that must be provided so that the firing pin can reach its maximum attainable velocity for which it has been designed, is given by Eq.(E-16):

$$L_t = \frac{4m_s gc^2}{\pi^2 \gamma D^2}\left(\frac{\tau_c \pi c}{KG}C_f + \frac{1}{c}\right) + 2\frac{D}{c} + L_p \tag{E-23}$$

where according to Eq.(C-14):

$$C_f = 2 + \frac{1}{\alpha}\left(3 - \frac{e^{-2\alpha}}{2} - 4e^{-\frac{e^{-2\alpha}}{2}}\right) \tag{E-24}$$

and

$$\alpha = \frac{m_s}{M} \tag{E-25}$$

Furthermore, the mean coil diameter D of the spring is related to the outside diameter D_0 of the spring and the firing pin by the following relationship:

$$D_0 = D + d = D(1 + \frac{1}{c})$$

therefore

$$D = \frac{D_0\, c}{1 + c} \tag{E-26}$$

The correct application of these design equations will now be discussed.

d. Optimization Procedure

Both the free length L_0 as well as the overall height of the system L_t, as given by Eq's.(E-17) and (E-23) respectively are directly as well as indirectly functions of the spring index c. Because of the complexity of this dependence, it is not practical to determine the shortest L_0 and L_t for a given condition analytically.

It has proven itself comparatively easy to search for that value of the spring index c which results in the shortest L_t under a given set of circumstances. Figure E-2 gives an example of such a computer search.

It was desired to find the shortest overall height L_t for a system employing I.D. No. 75 which has a nominal velocity v_N = 800 in/sec and a firing pin mass $M = 2.734 \times 10^{-6}$ lb-sec^2/in (refer to Figure D-2 in Appendix D and to Section 4a in the present appendix).

The outside diameter D_0 of the spring and the firing pin was chosen as 0.125 inches, and Figure E-1 indicated that the length of the firing pin had to be 0.304 inches in this case. The program was written to evaluate Eq's.(E-17), (E-18), (E-22) and (E-23) for L_0, m_s, N and L_t respectively in addition to computing the wire diameter d.

The above equations were computed for $3.4 \leq c \leq 6.00$. The shortest overall system can be found from the printout to correspond to c = 4.2 with a length of L_t = 1.713. The associated wire diameter is 0.02404 inches, the number of active turns are 30.1, and the free length of the spring is 1.277 inches.

The following additional parameters were used in the program:

γ = 0.283 lb/in^3 (density of steel)
C_D = 0.25, design correction factor (see Eq.(D-14))
τ_c = 200,000 psi, corrected maximum allowable shear stress for the music wire of the springs
g = 386.05 in/sec^2, acceleration of gravity
G = 11.5×10^{-6}, shear modulus of steel

Similar optimizations have been performed for all I.D. numbers and firing pin outside diameters indicated in Figure E-1.

Finally, all optimum possibilities with overall lengths less than 2.000 inches, which correspond to a certain I.D. number, were combined on one output sheet. Figure E-3 shows such a typical optimum design table for I.D. 75, (the same as used above). It now includes the least overall heights for such outside diameters as 0.200, 0.187, 0.170, 0.156, 0.140 and 0.125 inches.

IN-OZ = 14 VEL = 800.0 I.D. NO. = 75 M = 0.2734E-05

CD = 0.125 PINL = 0.304 C START = 3.4 DELCY = 0.1

SPRING INDEX(C)	SPRING MASS (MS) LB-SEC*SEC/IN*E6	WIRE DIAM (SMALLD) (INCHES)	ACTIVE TURNS (NACT)	FREE LENGTH OF SPRING (LZERO) (INCHES)	EXPANDED SPRING PLUS PIN (OVRALL) (INCHES)
3.4	5.2203	0.02841	37.0	1.553	2.077
3.5	4.6349	0.02778	34.2	1.435	1.928
3.6	4.2497	0.02717	32.5	1.367	1.842
3.7	3.9666	0.02660	31.5	1.325	1.785
3.8	3.7454	0.02604	30.9	1.299	1.753
3.9	3.5656	0.02551	30.4	1.283	1.731
4.0	3.4154	0.02500	30.2	1.276	1.719
4.1	3.2873	0.02451	30.1	1.274	1.715
4.2	3.1762	0.02404	30.1	1.277	1.713
4.3	3.0787	0.02358	30.2	1.283	1.717
4.4	2.9922	0.02315	30.3	1.293	1.725
4.5	2.9146	0.02273	30.5	1.306	1.737
4.6	2.8450	0.02232	30.7	1.322	1.751
4.7	2.7815	0.02193	31.0	1.340	1.768
4.8	2.7238	0.02155	31.3	1.360	1.787
4.9	2.6708	0.02119	31.7	1.382	1.809
5.0	2.6220	0.02083	32.1	1.406	1.832
5.1	2.5769	0.02049	32.5	1.431	1.858
5.2	2.5350	0.02016	32.9	1.459	1.885
5.3	2.4961	0.01984	33.3	1.488	1.914
5.4	2.4597	0.01953	33.8	1.518	1.944
5.5	2.4256	0.01923	34.3	1.550	1.977
5.6	2.3937	0.01894	34.8	1.583	2.010
5.7	2.3636	0.01866	35.3	1.618	2.045
5.8	2.3353	0.01838	35.8	1.655	2.082
5.9	2.3086	0.01812	36.4	1.692	2.120
6.0	2.2833	0.01786	36.9	1.731	2.160

Figure E-2

Optimization of Overall Length of System with Respect to Spring Index c

IN-OZ = 14 I.D. NO. = 75
VELOCITY = 800.0 PIN MASS = 2.734

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.200	.119	3.0787	.03774	7.4	0.558	0.608	0.728	.353
.187	.135	3.0787	.03528	9.0	0.623	0.680	0.816	.389
.170	.164	3.0787	.03208	12.0	0.733	0.801	0.967	.449
.156	.195	3.0787	.02943	15.5	0.853	0.934	1.131	.516
.140	.242	3.0787	.02642	21.5	1.038	1.139	1.384	.620
.125	.304	3.0787	.02358	30.2	1.283	1.409	1.717	.759

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

Figure E-3

Typical Optimum Design Table

(When comparing Figures E-2 and E-3 it can be seen that Figure E-3 shows for an outside diameter of 0.125 inches the values which follow directly below the minimum value in Figure E-2. Thus Figure E-2 gives a minimum overall height of 1.713 inches, while Figure E-3 gives 1.717 as the minimum. This discrepency has its reason in the needs of the final program. The error is of course insignificant.)

Figure E-3 also lists all other necessary design information such as: wire diameter, number of active turns, free length of spring, as well as its solid height, (which takes two inactive turns into account).

In addition, the spring mass is printed out for checking purposes. (The value of the spring masses are very close for all designs for a given I.D. number since the spring index at optimum is essentially identical for most. (See Eq.(E-18) for dependency on c.))

Appendix F gives all optimum design tables. Their information furnishes all those systems which are smaller than 2.000 inches in overall height, and assure 100 percent firing for the M42G primer.

APPENDIX F

OPTIMUM DESIGN TABLES

The present appendix lists approximately 215 spring striker systems with overall length of less than 2.000 inches and outside diameters of less than 0.375 inches. (See Appendix E for background.) All systems correspond to actual test points and 100 percent firing is assured in all cases. (Note the I.D. numbers as well as their associated energy levels, firing pin masses, and nominal firing pin velocities [v_N].) As in Figure E-3 of Appendix E the following complete design information is given:

1. Outside diameter of the system
2. Length of firing pin (assumed to be a solid cylinder without consideration of hemispherical tip of 0.032 inches)
3. Mass of the spring
4. Wire diameter of the spring
5. Active number of turns
6. Free length of spring (based on assumption of two inactive turns turns of spring)
7. Solid height of spring (again including two inactive turns)
8. Expanded length of spring at maximum velocity of firing pin
9. Overall height of system

Similar to the tabulations of Figures E-2 and E-3 in Appendix E, the following values were used in the computations:

γ = 0.283 lb/in^3 (density of steel music wire)
C_D = 0.25, design correction factor for velocity (see Eq.(D-14) in Appendix D)
τ_c = 200,000 psi, corrected maximum shear stress of spring at solid height
g = 386.05 in/sec^2, acceleration of gravity
G = 11.5x10^6 psi, shear modulus of steel

The computer program, (see sample printout below), was written in MAD (Michigan Algorithm Decoder), and all computations were made on an IBM 7040 computer at the City College Computation Center of the City University of New York.

a. Computer Program

```
********************
SPRING OPTIMIZATION
********************
C = MEAN COIL DIAMETER/WIRE DIAMETER
D = MEAN COIL DIAMETER
DELCY = INCREMENT OF C
FTT = LENGTH OF EXPANDED SPRING AT INSTANT OF MASS SEPARATION
K = STRESS CONCENTRATION OF SPRING
LZERO = FREE LENGTH OF SPRING
M = MASS OF FIRING PIN
MS = MASS OF SPRING
NACT = NUMBER OF ACTIVE COILS
NTOT = TOTAL NUMBER OF TURNS
OD = OUTER DIAMETER OF SPRING
OVRALL = OVERALL REQUIRED LENGTH
PINL = LENGTH OF FIRING PIN
SMALLD = WIRE DIAMETER
SOLH = SOLID HEIGHT
VEL = EXPERIMENTAL VELOCITY

 INTEGER INOZ, ID, ID1
 EXECUTE NOHD.
 READ DATA
 ME6 = M*1.0E6
THE LAST DATA CARD MUST BE J = 1-THIS IS A SIGNAL TO THE
PROGRAM THAT ALL THE DATA HAS BEEN READ
 INTEGER J
 WHENEVER J .E. 1
 PRINT FORMAT NOTES1
 PRINT FORMAT NOTES2
 PRINT FORMAT NOTES3
 TRANSFER TO START
 END OF CONDITIONAL
 WHENEVER (ID .NE. ID1) .AND. (ID1 .NE. 0)
    PRINT FORMAT NOTES1
    PRINT FORMAT NOTES2
    PRINT FORMAT NOTES3
 END OF CONDITIONAL
 WHENEVER ID .NE. ID1
    PRINT FORMAT DATA1,INOZ,ID,VEL,ME6
    PRINT FORMAT HDNG1
    PRINT FORMAT HDNG2
    PRINT FORMAT HDNG3
 END OF CONDITIONAL
 VECTOR VALUES NOTES1= $1H ///1H H*TOTAL TURNS OF SPRING ARE OBTAINED BY
1  ADDING 2 TURNS*/1H H*TO NUMBER OF ACTIVE TURNS**$
 VECTOR VALUES NOTES2 = $1HOH*ALL UNITS OF LENGTH ARE IN INCHES*/1HOHJAL
1 L UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6J/1HOH*ALL UNITS OF VE
1 LOCITY ARE IN IN/SEC**$
 VECTOR VALUES NOTES3 = $1HOH*OVERALL HEIGHT OF SYSTEM IS OBTAINED BY AD
```

```
 1 DING EXPANDED LENGTH OF SPRING*/1H H*TO LENGTH OF PIN**$
   VECTOR VALUES DATA1 = $1H1H*IN-OZ = *I2,S10,H*I.D. NO. = *I3/1H H*VELOC
 1 ITY = *F6.1,S3,H*PIN MASS = *F6.3*$
   VECTOR VALUES HDNG1 = $1HOH*O.D.*S3,H*PIN*S4,H*SPRING*S3,H*WIRE*S3,
 1 H*ACTIVE*S3,H*FREE*S3,H*EXPANDED*S4,H*OVERALL*S4,
 2 H*SOLID**$
   VECTOR VALUES HDNG2 =$1H S6,H*LENGTH*S3,H*MASS*S4,H*DIAM*S3,H*TURNS*
 1 S3,H*LENGTH*S3,H*LENGTH*2(S5,H*HEIGHT*)*$
   VECTOR VALUES HDNG3 = $1H S46,H*OF SPRING*S2,H*OF SYSTEM*S2,H*OF SPRING
 1 **$
   FTT = 100.
   FTTP = 200.
   THROUGH GAMMA, FOR C = CST, DELCY, C .G. CFIN
   WHENEVER FTTP .L. FTT
   PRINT FORMAT ANS, OD, PINL, MSE6, SMALLD, NACT, LZERO, FTTP, OVRALL,
 1    SOLH
      TRANSFER TO NEXT
   END OF CONDITIONAL
   VECTOR VALUES ANS = $1H F4.3,S3,F4.3,S3,F6.4,S2,F6.5,S1,F5.1,S3,F6.3,
 1 S3,F6.3,S6,F6.3,S6,F4.3*$
   FTTP = FTT
   V = VEL*1.25
   D = (OD*C)/(1.0 + C)
   K = (4.0*C - 1.0)/(4.0*C - 4.0) + 0.615/C
   MS = -0.5*M*ELOG.(-2.0*ELOG.(V*K/3053.435 + 0.5))
   MSE6 = MS*1.0E6
   ALPHA = MS/M
   KF = 2.0 + (1.0/ALPHA)*(3.0 - 0.5*EXP.(-2.0*ALPHA)
 1      - 4.0*EXP.(-0.5*EXP.(-2.0*ALPHA)))
   FTT =(552.8637*C*C*MS)/(D*D)*((0.05463*C*KF)/K + 1.0/C)
 1        + 2.0*D/C
   SMALLD = D/C
   NACT =552.8637*MS/(D*SMALLD*SMALLD)
   NTOT = NACT + 2.0
   LZERO =(552.8637*C*C*MS)/(D*D)*((0.05463*C)/K
 1         + 1.0/C) + 2.0*D/C
   OVRALL = FTT + PINL
   SOLH = SMALLD*NTOT
   CONTINUE
   ID1 = ID
   TRANSFER TO START
   END OF PROGRAM
```

b. Optimum Design Tables

IN-OZ = 16 I.D. NO. = 46
VELOCITY = 400.0 PIN MASS = 12.500

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.375	.154	2.6986	.06944	1.0	0.264	0.264	0.418	.209
.312	.222	2.9459	.06367	1.6	0.291	0.289	0.512	.230
.250	.348	3.2160	.05556	3.0	0.350	0.348	0.696	.276
.218	.454	3.3873	.05070	4.4	0.409	0.405	0.860	.322
.200	.543	3.3873	.04651	5.6	0.459	0.454	0.998	.355
.187	.618	3.4850	.04452	6.8	0.504	0.498	1.117	.393
.170	.752	3.4850	.04048	9.1	0.583	0.576	1.329	.448
.156	.890	3.5924	.03805	11.6	0.668	0.661	1.551	.519

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 16 I.D. NO. = 53
VELOCITY = 500.0 PIN MASS = 9.000

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.250	.222	3.2735	.05556	3.0	0.355	0.351	0.573	.279
.218	.291	3.3560	.04955	4.5	0.415	0.411	0.702	.321
.200	.348	3.4465	.04651	5.7	0.465	0.460	0.808	.360
.187	.395	3.4465	.04349	7.0	0.512	0.506	0.902	.392
.170	.481	3.5463	.04048	9.2	0.591	0.586	1.067	.455
.156	.570	3.5463	.03714	12.0	0.680	0.673	1.244	.518
.140	.709	3.5463	.03333	16.5	0.819	0.809	1.520	.618
.125	.890	3.5463	.02976	23.2	1.004	0.991	1.883	.751

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 16 I.D. NO. = 60
VELOCITY = 600.0 PIN MASS = 5.555

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.250	.154	3.2956	.05435	3.2	0.363	0.364	0.518	.260
.218	.202	3.3822	.04844	4.7	0.428	0.429	0.632	.325
.200	.241	3.3822	.04444	6.1	0.482	0.483	0.725	.359
.187	.275	3.4781	.04250	7.4	0.531	0.534	0.809	.398
.170	.334	3.4781	.03864	9.8	0.616	0.620	0.954	.456
.156	.396	3.4781	.03545	12.7	0.711	0.714	1.112	.521
.140	.493	3.5848	.03256	17.4	0.855	0.862	1.355	.632
.125	.618	3.5848	.02907	24.4	1.048	1.057	1.676	.769
.110	.798	3.5848	.02558	35.9	1.330	1.341	2.140	.969

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 16 I.D. NO. = 67
VELOCITY = 700.0 PIN MASS = 4.081

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.218	.148	3.3964	.04542	5.3	0.463	0.478	0.626	.330
.200	.177	3.3964	.04167	6.8	0.525	0.542	0.721	.368
.187	.202	3.4957	.03973	8.3	0.590	0.602	0.805	.410
.170	.245	3.4957	.03617	11.0	0.674	0.704	0.950	.472
.156	.291	3.4957	.03319	14.3	0.786	0.816	1.109	.541
.140	.362	3.4957	.02973	19.8	0.953	0.990	1.355	.648
.125	.454	3.4957	.02660	27.8	1.174	1.220	1.678	.792

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 16
VELOCITY = 800.0   I.D. NO. = 74
PIN MASS = 3.125

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.200	.136	3.5190	.03774	8.4	0.627	0.684	0.821	.393
.187	.154	3.5190	.03528	10.3	0.702	0.767	0.922	.434
.170	.188	3.5190	.03208	13.7	0.828	0.906	1.096	.504
.156	.223	3.5190	.02943	17.7	0.966	1.059	1.284	.581
.140	.277	3.5190	.02642	24.5	1.179	1.294	1.574	.701
.125	.348	3.5190	.02358	34.5	1.460	1.603	1.956	.861

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 16 I.D. NO. = 81
VELOCITY = 900.0 PIN MASS = 2.469

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.200	.107	3.5957	.03125	12.1	0.907	1.078	1.185	.434
.187	.122	3.5957	.02922	14.8	1.024	1.219	1.343	.490
.170	.148	3.5957	.02656	19.6	1.222	1.458	1.606	.575
.156	.176	3.5957	.02438	25.4	1.437	1.716	1.896	.668

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 14 I.D. NO. = 47
VELOCITY = 400.0 PIN MASS = 10.938

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.312	.195	2.5287	.06240	1.4	0.270	0.269	0.465	.215
.250	.304	2.8142	.05556	2.6	0.320	0.318	0.622	.255
.218	.397	2.8859	.04955	3.9	0.371	0.368	0.765	.290
.200	.475	2.9640	.04651	4.9	0.413	0.409	0.684	.323
.187	.541	3.0495	.04452	6.0	0.452	0.448	0.989	.355
.170	.658	3.0495	.04048	7.9	0.520	0.514	1.173	.403
.156	.779	3.1435	.03805	10.2	0.594	0.588	1.367	.463
.140	.970	3.1435	.03415	14.1	0.711	0.703	1.674	.549

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 14 I.D. NO. = 54
VELOCITY = 500.0 PIN MASS = 7.000

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.250	.195	2.7983	.05435	2.7	0.325	0.321	0.516	.254
.218	.254	2.9365	.04955	3.9	0.376	0.372	0.626	.294
.200	.304	2.9365	.04545	5.1	0.420	0.414	0.719	.322
.187	.346	3.0156	.04349	6.1	0.459	0.454	0.801	.354
.170	.421	3.0156	.03953	8.2	0.529	0.523	0.945	.402
.156	.499	3.1030	.03714	10.5	0.605	0.598	1.098	.463
.140	.621	3.1030	.03333	14.5	0.725	0.717	1.339	.549
.125	.779	3.1030	.02976	20.3	0.886	0.875	1.656	.665

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 14 I.D. NO. = 61
VELOCITY = 600.0 PIN MASS = 4.861

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.250	.135	2.8150	.05319	2.8	0.332	0.332	0.467	.255
.218	.177	2.8839	.04739	4.2	0.388	0.388	0.566	.292
.200	.211	2.9597	.04444	5.3	0.433	0.434	0.646	.326
.187	.240	2.9597	.04156	6.5	0.477	0.478	0.719	.354
.170	.292	3.0435	.03864	8.6	0.549	0.552	0.845	.409
.156	.346	3.0435	.03545	11.1	0.631	0.634	0.981	.465
.140	.431	3.0435	.03182	15.4	0.759	0.763	1.195	.552
.125	.541	3.1370	.02907	21.4	0.925	0.933	1.474	.680
.110	.698	3.1370	.02558	31.4	1.170	1.180	1.879	.854

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 14 I.D. NO. = 68
VELOCITY = 700.0 PIN MASS = 3.571

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.218	.130	2.9720	.04542	4.6	0.416	0.430	0.560	.300
.200	.155	2.9720	.04167	6.0	0.470	0.485	0.641	.332
.187	.177	3.0588	.03979	7.3	0.518	0.537	0.714	.368
.170	.215	3.0588	.03617	9.7	0.603	0.625	0.841	.422
.156	.254	3.0588	.03319	12.5	0.696	0.723	0.978	.461
.140	.317	3.0588	.02979	17.3	0.841	0.874	1.193	.575
.125	.397	3.0588	.02660	24.3	1.034	1.075	1.474	.699

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 14 I.D. NO. = 75
VELOCITY = 800.0 PIN MASS = 2.734

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.200	.119	3.0787	.03774	7.4	0.558	0.608	0.728	.353
.187	.135	3.0787	.03528	9.0	0.623	0.680	0.816	.389
.170	.164	3.0787	.03208	12.0	0.733	0.801	0.967	.449
.156	.195	3.0787	.02943	15.5	0.853	0.934	1.131	.516
.140	.242	3.0787	.02642	21.5	1.038	1.139	1.384	.620
.125	.304	3.0787	.02358	30.2	1.283	1.409	1.717	.759

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 14 I.D. NO. = 82
VELOCITY = 900.0 PIN MASS = 2.160

I.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.187	.107	3.1457	.02922	12.9	0.904	1.074	1.183	.436
.170	.130	3.1457	.02656	17.2	1.076	1.282	1.414	.510
.156	.154	3.1457	.02438	22.2	1.263	1.508	1.665	.591
.140	.192	3.1457	.02188	30.8	1.552	1.855	2.051	.717
.125	.240	3.1457	.01953	43.2	1.939	2.311	2.556	.883
.110	.310	3.1457	.01719	63.4	2.477	2.968	3.285	****
.100	.376	3.1457	.01563	84.4	2.987	3.580	3.965	****
.093	.429	3.1457	.01453	105.0	3.446	4.133	4.572	****

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 12 I.D. NO. = 48
VELOCITY = 400.0 PIN MASS = 9.375

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.312	.157	2.1280	.06118	1.3	0.250	0.249	0.416	.199
.250	.261	2.3554	.05435	2.3	0.291	0.289	0.550	.231
.218	.340	2.4735	.04955	3.3	0.332	0.330	0.670	.263
.200	.407	2.5405	.04651	4.2	0.367	0.364	0.771	.290
.187	.463	2.5405	.04349	5.2	0.401	0.396	0.860	.312
.170	.564	2.6138	.04048	6.8	0.457	0.453	1.017	.357
.155	.668	2.6138	.03714	8.8	0.521	0.515	1.184	.402
.140	.831	2.6943	.03415	12.1	0.619	0.613	1.444	.480
.125	****	2.6943	.03049	17.0	0.752	0.743	1.788	.578

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 12 I.D. NO. = 55
VELOCITY = 500.0 PIN MASS = 6.000

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.250	.167	2.3466	.05319	2.3	0.294	0.291	0.458	.230
.218	.218	2.4552	.04844	3.4	0.337	0.333	0.551	.262
.200	.261	2.5170	.04545	4.4	0.373	0.369	0.630	.289
.187	.297	2.5848	.04349	5.3	0.406	0.402	0.699	.316
.170	.361	2.5848	.03953	7.0	0.465	0.460	0.821	.356
.156	.427	2.6597	.03714	9.0	0.529	0.524	0.951	.407
.140	.532	2.6597	.03333	12.4	0.631	0.624	1.157	.480
.125	.667	2.6597	.02976	17.4	0.766	0.758	1.427	.578
.110	.862	2.7428	.02683	25.3	0.963	0.954	1.816	.733

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 12   I.D. NO. = 62
VELOCITY = 600.0   PIN MASS = 4.167

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.218	.151	2.4721	.04739	3.6	0.346	0.346	0.498	.264
.200	.181	2.5371	.04444	4.6	0.384	0.385	0.566	.292
.187	.206	2.5371	.04156	5.6	0.420	0.422	0.628	.315
.170	.251	2.6090	.03864	7.4	0.482	0.485	0.736	.361
.156	.297	2.6090	.03545	9.5	0.551	0.554	0.852	.408
.140	.369	2.6090	.03182	13.2	0.660	0.663	1.033	.483
.125	.463	2.6891	.02907	18.3	0.801	0.808	1.271	.591
.110	.598	2.6891	.02556	26.9	1.010	1.019	1.618	.740
.100	.724	2.6891	.02326	35.8	1.207	1.217	1.942	.880
.093	.828	2.6891	.02163	44.5	1.385	1.396	2.226	****

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 12 I.D. NO. = 69
VELOCITY = 700.0 PIN MASS = 3.061

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.200	.133	2.5475	.04167	5.1	0.415	0.428	0.562	.297
.187	.151	2.5475	.03896	6.3	0.457	0.472	0.624	.322
.170	.184	2.6220	.03617	8.3	0.527	0.547	0.731	.372
.156	.218	2.6220	.03319	10.7	0.606	0.629	0.848	.422
.140	.271	2.6220	.02979	14.8	0.730	0.758	1.031	.501
.125	.340	2.6220	.02660	20.8	0.894	0.929	1.271	.607
.110	.440	2.6220	.02340	30.6	1.132	1.177	1.621	.762
.100	.532	2.7055	.02174	40.4	1.349	1.410	1.942	.923

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 12 I.D. NO. = 76
VELOCITY = 800.0 PIN MASS = 2.344

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.200	.102	2.6395	.03774	6.3	0.489	0.533	0.635	.314
.187	.116	2.6395	.03528	7.7	0.544	0.593	0.710	.343
.170	.141	2.6395	.03208	10.3	0.637	0.696	0.838	.394
.156	.167	2.6395	.02943	13.3	0.739	0.809	0.978	.451
.140	.208	2.6395	.02642	18.4	0.898	0.984	1.194	.539
.125	.261	2.6395	.02358	25.9	1.107	1.215	1.479	.657
.110	.337	2.7231	.02115	37.9	1.402	1.549	1.886	.843

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 12 I.D. NO. = 83
VELOCITY = 900.0 PIN MASS = 1.852

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.170	.111	2.6971	.02656	14.7	0.930	1.107	1.220	.444
.156	.132	2.6971	.02438	19.1	1.090	1.300	1.434	.514
.140	.162	2.6971	.02188	26.4	1.337	1.597	1.762	.621

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 10 I.D. NO. = 49
VELOCITY = 400.0 PIN MASS = 7.813

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.312	.139	1.7139	.05887	1.1	0.228	0.228	0.367	.181
.250	.217	1.9193	.05319	1.9	0.260	0.259	0.476	.208
.218	.264	2.0101	.04844	2.8	0.294	0.291	0.576	.232
.200	.339	2.0614	.04545	3.6	0.322	0.319	0.658	.253
.187	.386	2.1172	.04349	4.3	0.348	0.345	0.731	.275
.170	.470	2.1783	.04048	5.7	0.394	0.391	0.861	.311
.156	.556	2.1783	.03714	7.3	0.447	0.442	0.998	.347
.140	.693	2.2454	.03415	10.1	0.528	0.522	1.215	.412
.125	.869	2.2454	.03049	14.1	0.637	0.630	1.500	.492

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 10 I.D. NO. = 56
VELOCITY = 500.0 PIN MASS = 5.000

I.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.250	.139	1.9155	.05208	2.0	0.264	0.260	0.400	.207
.218	.182	1.9988	.04739	2.9	0.298	0.294	0.476	.231
.200	.217	2.0460	.04444	3.7	0.327	0.323	0.540	.253
.187	.247	2.0975	.04250	4.4	0.354	0.350	0.597	.274
.170	.301	2.1540	.03953	5.8	0.401	0.397	0.698	.310
.156	.356	2.1540	.03628	7.6	0.455	0.449	0.806	.347
.140	.443	2.2164	.03333	10.3	0.537	0.532	0.975	.411
.125	.556	2.2164	.02976	14.5	0.650	0.642	1.199	.492
.110	.718	2.2164	.02619	21.3	0.814	0.804	1.524	.611
.100	.869	2.2856	.02439	28.1	0.965	0.957	1.826	.734
.093	.994	2.2856	.02268	34.9	1.105	1.095	2.089	.839

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 10 I.D. NO. = 63
VELOCITY = 600.0 PIN MASS = 3.472

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.218	.126	2.0106	.04638	3.0	0.305	0.305	0.431	.232
.200	.151	2.0598	.04348	3.8	0.335	0.336	0.487	.254
.187	.172	2.1140	.04156	4.7	0.364	0.366	0.538	.276
.170	.209	2.1140	.03778	6.2	0.416	0.417	0.627	.310
.156	.247	2.1739	.03545	7.9	0.471	0.474	0.721	.352
.140	.308	2.1739	.03182	11.0	0.560	0.563	0.872	.413
.125	.386	2.1739	.02841	15.4	0.680	0.683	1.071	.495
.110	.499	2.2406	.02558	22.4	0.850	0.858	1.357	.625
.100	.604	2.2406	.02326	29.3	1.014	1.022	1.627	.741
.093	.690	2.2406	.02163	37.1	1.161	1.171	1.862	.846

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 10 I.D. NO. = 70
VELOCITY = 700.0 PIN MASS = 2.551

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.200	.111	2.1231	.04167	4.3	0.359	0.371	0.482	.261
.187	.126	2.1231	.03896	5.2	0.394	0.407	0.533	.281
.170	.153	2.1851	.03617	6.9	0.451	0.468	0.621	.322
.156	.182	2.1851	.03319	8.9	0.516	0.536	0.718	.363
.140	.226	2.1851	.02979	12.4	0.618	0.642	0.869	.428
.125	.284	2.1851	.02660	17.4	0.754	0.783	1.069	.515
.110	.366	2.1851	.02340	25.5	0.952	0.989	1.358	.643
.100	.443	2.2547	.02174	33.7	1.132	1.182	1.625	.776
.093	.507	2.2547	.02022	41.9	1.299	1.357	1.864	.888

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 10 I.D. NO. = 77
VELOCITY = 800.0 PIN MASS = 1.953

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.187	.097	2.1992	.03528	6.4	0.465	0.506	0.604	.298
.170	.117	2.1992	.03208	8.6	0.542	0.591	0.709	.339
.156	.139	2.1992	.02943	11.1	0.626	0.684	0.824	.385
.140	.173	2.1992	.02642	15.3	0.757	0.829	1.004	.458
.125	.217	2.1992	.02358	21.6	0.930	1.020	1.240	.556
.110	.281	2.1992	.02075	31.6	1.182	1.297	1.582	.698
.100	.339	2.2689	.01923	42.0	1.410	1.557	1.896	.846

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 10   I.D. NO. = 84
VELOCITY = 900.0   PIN MASS = 1.543

I.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.170	.093	2.2471	.02656	12.3	0.784	0.931	1.026	.379
.156	.110	2.2471	.02438	15.9	0.916	1.091	1.203	.436
.140	.137	2.2471	.02188	22.0	1.121	1.338	1.478	.525
.125	.172	2.2471	.01953	30.9	1.390	1.662	1.838	.642

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 8 I.D. NO. = 71
VELOCITY = 700.0 PIN MASS = 2.041

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.187	.101	1.6986	.03896	4.2	0.331	0.341	0.442	.241
.170	.123	1.6986	.03542	5.6	0.377	0.389	0.512	.268
.156	.145	1.7483	.03319	7.1	0.426	0.442	0.587	.304
.140	.181	1.7483	.02979	9.9	0.506	0.526	0.707	.354
.125	.227	1.7483	.02660	13.9	0.614	0.637	0.866	.423
.110	.293	1.7483	.02340	20.4	0.771	0.801	1.096	.524
.100	.355	1.7483	.02128	27.1	0.918	0.955	1.312	.620
.093	.406	1.7483	.01979	33.7	1.052	1.094	1.503	.707

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 8 I.D. NO. = 78
VELOCITY = 800.0 PIN MASS = 1.562

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.170	.094	1.7589	.03208	6.9	0.446	0.486	0.580	.284
.156	.111	1.7589	.02943	8.9	0.512	0.559	0.671	.320
.140	.138	1.7589	.02642	12.3	0.616	0.674	0.813	.377
.125	.174	1.7589	.02358	17.2	0.753	0.825	1.002	.454
.110	.224	1.7589	.02075	25.3	0.954	1.046	1.273	.567
.100	.272	1.7589	.01887	33.7	1.141	1.253	1.529	.673
.093	.311	1.8146	.01788	41.8	1.304	1.440	1.751	.783

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

IN-OZ = 8 I.D. NO. = 85
VELOCITY = 900.0 PIN MASS = 1.234

C.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.156	.088	1.7971	.02438	12.7	0.743	0.883	0.972	.358
.140	.109	1.7971	.02188	17.6	0.905	1.079	1.190	.428
.125	.137	1.7971	.01953	24.7	1.120	1.337	1.477	.521
.110	.177	1.7971	.01719	36.2	1.430	1.710	1.891	.657

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING TO LENGTH OF PIN

IN-OZ = 6 I.D. NO. = 79
VELOCITY = 800.0 PIN MASS = 1.172

O.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.156	.083	1.3198	.02943	6.7	0.399	0.434	0.518	.255
.140	.104	1.3198	.02642	9.2	0.475	0.519	0.624	.296
.125	.130	1.3198	.02358	12.9	0.577	0.631	0.763	.352
.110	.168	1.3198	.02075	19.0	0.726	0.795	0.966	.435
.100	.204	1.3198	.01887	25.3	0.866	0.950	1.157	.514
.093	.233	1.3198	.01755	31.4	0.992	1.089	1.326	.586

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

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IN-OZ =  6              I.D. NO. =   86
VELOCITY =   900.0      PIN MASS =   0.926
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C.D.	PIN LENGTH	SPRING MASS	WIRE DIAM	ACTIVE TURNS	FREE LENGTH	EXPANDED LENGTH OF SPRING	OVERALL HEIGHT OF SYSTEM	SOLID HEIGHT OF SPRING
.140	.082	1.3486	.02188	13.2	0.690	0.821	0.904	.332
.125	.103	1.3486	.01953	18.5	0.850	1.013	1.118	.401
.110	.133	1.3486	.01719	27.2	1.081	1.292	1.428	.502
.100	.161	1.3486	.01563	36.2	1.298	1.553	1.718	.597
.093	.184	1.3486	.01453	45.0	1.494	1.789	1.977	.683

TOTAL TURNS OF SPRING ARE OBTAINED BY ADDING 2 TURNS
TO NUMBER OF ACTIVE TURNS

ALL UNITS OF LENGTH ARE IN INCHES

ALL UNITS OF MASS ARE IN LB-SEC*SEC/IN*E6

ALL UNITS OF VELOCITY ARE IN IN/SEC

OVERALL HEIGHT OF SYSTEM IS OBTAINED BY ADDING EXPANDED LENGTH OF SPRING
TO LENGTH OF PIN

END-OF-DATA ENCOUNTERED ON SYSTEM INPUT FILE.